Giant oscillations of the parallel-pumping threshold in magnetic materials with a biharmonic field modulation

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The change in the temporal phase of a paramagnetic magnon pair is determined by the same nonlinear equation as the change in the phase difference between the wave functions of paired superconducting electrons at a Josephson junction. Experiments with an rf modulation of the magnetic field during parallel microwave pumping of nucleus-like magnons in the antiferromagnet $CsMnF_3$ reveal anomalies in the behavior of the pumping threshold, as expected on the basis of this mathematical analogy.

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An rf modulation of the external magnetic field ($H = H_0 + H_m \cos \Omega t$) in experiments on the parametric excitation of magnons in magnetically ordered crystals increases the threshold value h_c of the microwave pump amplitude ($h_{11} = h \cos \omega_p t$). The situation most interesting for analysis is that of so-called fast modulation, in which the wave vector **k** of the excited magnon pair ω_k, ω_{-k} cannot change fast enough to satisfy the condition[1] $2\omega_k (H) = \omega_p$ [1]. The effect of the modulating field reduces to a modulation of the magnon frequency,

$$\omega_k = \omega_p/2 + (\partial \omega_k/\partial H)_{H_0} H_m \cos \Omega t \tag{1}$$

and thus to variations of the temporal phase Ψ_k of the magnon pair. The phase Ψ_k is defined as the sum of the phases ϕ_k and ϕ_{-k} of the individual magnons of the pair, and it is simply the phase shift of the longitudinal-magnetization waves of the sample, $\tilde{\mu}_{\parallel} = -\mu_a \cos(\omega_p t + \Psi_k)$, with respect to the pump. It is easily shown that the amplitude μ_a is proportional to the occupation numbers n_k for parametric magnons, whose evolution is described in a first approximation with $h \approx h_c$ by the equations[2]

$$\dot{n}_n/n_k = -2\gamma_k + 2hV_k \sin \Psi_k , \tag{2a}$$

$$\dot{\Psi}_k = (2\omega_k - \omega_p) + 2hV_k \cos \Psi_k . \tag{2b}$$

From (2a) with $\dot{n}_k = 0$ we find the familiar relation $h_c = h_{c0}/\sin\Psi_k$, and where $h_{c0} \equiv \gamma_k/V_k$, γ_k is the magnon relaxation frequency, and V_k describes the coupling of the pair with the pump. The presence of the factor $\sin \psi_k$ in (2a) has been verified experimentally.[3] The nonlinear term $2hV_k \cos\Psi_k$ makes the second equation in (2) nontrivial; *if this equation is correct*, this term could have some interesting physical consequences.

We set $\theta \equiv \pi/2 - \Psi_k$, and we rewrite (2b) as

$$d\theta/d\tau + \sin\theta = \alpha\cos\omega\tau, \tag{3}$$

where $\tau \equiv 2hV_k t$; $\omega \equiv \Omega/2hV_k$; $\alpha \equiv -(\partial\omega_k/\partial H)H_m/hV_k$. For a weak modulation, i.e., $|\alpha| \ll \sqrt{1+\omega^2}$, with $\theta \ll 1$, we find

$$h_c/h_{co} = \langle \sin\Psi_k \rangle^{-1} = \langle \cos\theta \rangle^{-1} \approx 1 + H_m^2(\partial\omega_k/\partial H)^2/(\Omega^2 + 4\gamma_k^2), \tag{4}$$

i.e., an expression derived in Ref. 1 and checked in Refs. 1 and 4 (the angle brackets denote the average over a time $t \gg 2\pi/\Omega$). If the condition for a slight modulation is not satisfied, specifically, if

$$H_m \gtrsim \sqrt{h^2 + \Omega^2/4V_k^2}\,/2 \tag{5}$$

then the dependence $\theta(t)$ and thus $h_c(\Omega)$ may turn out to be extremely interesting. The situation is even more interesting because the nonlinear equation in (3) is the same as the equation for the phase difference betwen the wave functions of paired superconducting electrons on the two sides of a Josephson junction[5] (without a bias voltage) subjected to a harmonic voltage. In particular, if a *biharmonic* voltage is applied to a junction with a bias voltage V, the voltage–current characteristic of the junction will exhibit a large number of clearly defined features at the bias voltages $V = \hbar(n_1\Omega_1 \pm n_2\Omega_2)/2e$, which are determined by sum and difference frequencies.[6]

Condition (5) can be satisfied most simply in the parallel pumping of nucleuslike magnons in an easy-plane antiferromagnet ($CsMnF_3$, for example; see Ref. 7), since for these magnons we have $\gamma_k/2\pi \sim 10^4$ Hz and $2V_k = \partial\omega_k/\partial H \sim \gamma_e/25$, where γ_e is the magnetomechanical ratio for the electron. We have accordingly carried out an experimental study of the effect of a *biharmonic* field modulation on the threshold pump amplitude for a $CsMnF_3$ single crystal over the temperature range 1.9–4.2 K with continuous pumping at the frequency $\omega_p/2\pi = 1000$ MHz. The sample was placed at the center of a spiral half-wave transit cavity 6 mm in diameter with a loaded $Q \sim 300$. The cavity was coupled with the conducting coaxial cables by rods at opposite ends of the spiral. A biharmonic modulation of the magnetic field at the frequencies F_1 *and* F_2 ($F = \Omega/2\pi$) was produced by an external coil $\sim$2 cm in diameter, through which currents were driven by two sonic-frequency oscillators in a parallel connection. The threshold for the parametric excitation of the nuclear magnons was determined within a relative error of 1% from the detected signal by means of a sensitive spectrum analyzer.

To measure the amplitudes H_{m1} and H_{m2}, we connected a standard resistor in series with the modulation coil and displayed the output signal from this resistor on the screen of a second spectrum analyzer. We found no spectral components other than F_1 and F_2 in this signal, so we conclude that there is no significant nonlinearity in the modulation system.

We measured the relative increase in the threshold for the parametric excitation of nuclear magnons as a function of the frequency F_1 of the first modulating field at fixed values of H_{m1}, H_{m2}, and F_2. Figure 1 shows how the frequency dependence of the relative increase in the threshold changes with increasing H_{m2}. The positions of the many deep minima can be described surprisingly well by the expression $n_1F_1 = n_2F_2$,

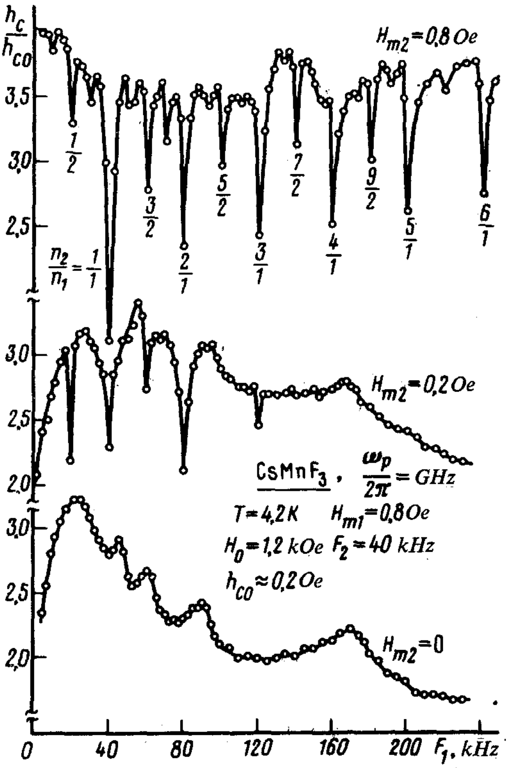

FIG. 1. Dependence of the relative increase in the threshold, h_c/h_{co} ($h_{co}\approx 0.2$ OE is the threshold in the absence of modulation), on the frequency F_1 of the first modulating field and at a fixed modulation amplitude $H_{m1}=0.8$ Oe of the first field. The constant field $H_0=1.2$ kOe is applied parallel to the easy plane of the $CsMnF_3$ signal crystal. The temperature is $T=4.2$ K. The peaks on the lower curve are interpreted in Ref. 7.

where n_1 and n_2 are integers. The corresponding results for $F_2=120$ kHz reveal minima at subharmonics of the frequency F_2 down to the eighth ($n_2/n_1=\frac{1}{8}$). The depth of these minima changes only insignificantly upon changes in the temperature and the constant field.

The giant oscillations of the threshold observed here with a biharmonic field modulation indicate a very nonlinear nature of the equation for the phase of the parametric magnon pair. A quantitative test of this equation will require calculating the depths of the minima.

[1]V. V. Zautkin, V. S. L'vov, B. N. Orel, and S. Starobinets, Zh. Eksp. Teor. Fiz. **72**, 272 (1977) [Sov. Phys. JETP **45**, 143 (1977)].

[2]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1975)].

[3]L. A. Prozorova and A. I. Smirnov, Zh. Eksp. Teror. Fiz. **67**, 1952 (1974) [Sov. Phys. JETP **40**, 970 (1974)].

[4]V. I. Ozhogin, A. Yu. Yakubovskiĭ, A. V. Abryutin, and S. M. Suleĭmanov, Zh. Eksp. Teor. Fiz. **77**, 2061

(1979) [Sov. Phys. JETP **50**, 984 (1979)].
[5]K. K. Likharev and B. T. Ul'rikh, Sistemy s dzhozefsonovskimi kontaktami (Josephson–Junction Systems), Izd. MGU, 1978 p. 83.
[6]A. Longacre and S. Shapiro, in: Superconductive Tunneling and Applications (ed. L. Solymar), Halsted, New York, 1972 (Russ. transl. Mir, Moscow, 1974).
[7]V. I. Ozhogin, S. M. Suleĭmanov, and A. Yu. Yakubovskii, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 308 (1980) [JETP Lett. **32**, 284 (1980)].

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