Stochasticity of classical Yang-Mills mechanics and its elimination by using the Higgs mechanism

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The Higgs mechanism eliminates stochasticity of the Yang-Mills classical mechanics observed by Matinyan, Savvidi, and Ter-Arutyunyan-Savvidi {Zh. Eksp. Teor. Fiz. **80**, 830 (1981) [Sov. Phys. JETP. **53**, 421 (1981)]}. The phase transition occurs at the critical value of the parameter ($\pi_c \approx 0.15$), which characterizes the Yang-Mills-Higgs system.

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The Yang–Mills classical equations in Minkowski space without external sources, in which the vector potential $A_\mu^a(x)$ in a certain coordinate system depends solely on time (see Refs. 1 and 2), were studied in our previous paper.[1] The system described by such potentials $A_\mu^a(t)$ reduces to a discrete nonlinear mechanical system of equations (Yang–Mills classical mechanics) with the Hamiltonian[1]

$$H_{\rm YM} = \Sigma \frac{1}{2}(\dot{A}_i^a)^2 + \frac{g^2}{4}[(A_i^a A_i^a)^2 - (A_i^a A_j^a)^2] \tag{1}$$

and with the coupling equations

$$M^a \equiv \epsilon^{abc} A_i^b \dot{A}_i^c = 0. \tag{2}$$

The system of equations (1) has nine degrees of freedom ($a, i = 1,2,3$) and four remaining integrals: $H_{\rm YM}, M_i = \epsilon_{ijk} A_j^a A_k^a$ for the *SU(2)* group. As the size of the gauge group is increased, the number of "missing" integrals for the *SU(N)* group increases as $2N^2 - 6$.

A strong instability of the trajectories in the two- and three-dimensional subsystems (1) with the potentials $U_2 = g^2/2(A_1^1 A_2^2)^2$,

$$U_3 = \frac{g^2}{2}[(A_1^1 A_2^2)^2 + (A_1^1 A_3^3)^2 + (A_2^2 A_3^3)^2],$$

led us to conclude that they are stochastic.[1] This conclusion was subsequently confirmed.[3] A characteristic property of the equations analyzed by us[1] is that they also have a denumerable set of periodic trajectories.

The inclusion of a phase (the confinement-disorder phase or the Higgs-order phase) in the gauge theories[4] has recently been a focus of attention. We shall assume that the disorder phase [e.g., that in Eq. (1)] corresponds to the case in which the complete set of isolating integrals is missing in a classical system and that the order phase corresponds to the systems with a complete set of isolating integrals (the case in which they are equal to the number of degrees of freedom.

In view of this, it would be of interest to investigate the phases of the classical gauge systems with spin symmetry breaking.

We shall analyze the gauge theory with isodoublet violation of the $SU(2)$ group in the gauge $A_0^a = 0$. The Hamiltonian corresponding to Eq. (1) now has the form

$$H = H_{\text{YM}} + \frac{1}{2}(\dot{B}_a^2 + \dot{\sigma}^2) + \frac{g^2}{4}(A_i^a A_i^a)$$

$$\times \left[\frac{B_a^2}{2} + \left(\frac{\sigma}{\sqrt{2}} + \eta\right)^2\right] + \lambda^2 \left[\frac{B_a^2}{2} + \left(\frac{\sigma}{\sqrt{2}} + \eta\right)^2 - \eta^2\right]^2, \tag{3}$$

and the coupling equations can be written as follows:

$$\epsilon^{abc} A_i^b \dot{A}_i^c - \frac{\eta}{\sqrt{2}} \dot{B}_a + \frac{1}{2}\left[\sigma \dot{B}_a - B_a \dot{\sigma} - \epsilon^{abc} B_b \dot{B}_c\right] = 0, \tag{4}$$

where η is the vacuum expectation value of the scalar field ϕ:

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} iB_1 + B_2 \\ \sqrt{2}\eta + \sigma - iB_3 \end{pmatrix}.$$

We shall carefully analyze the two-dimensional case (see Ref. 1), in which $A_1^1 \equiv x$ and $A_2^2 \equiv y$, and all the remaining components A_i^a, B_a, and σ are equal to zero

$$H \equiv \mu^4 = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + \frac{g^2}{2}(xy)^2 + \frac{g^2\eta^2}{4}(x^2 + y^2). \tag{5}$$

The motion of the system (5) is characterized by a single parameter

$$\pi = \left(\frac{g}{2}\right)^2 \left(\frac{\eta}{\mu}\right)^4, \tag{6}$$

which can easily be verified by means of the transformation $x \to \alpha x$, $y \to \alpha y$, and $t \to \beta t$.

Our goal in this letter is to calculate the critical parameter π_c for the phase transition; the calculation of this parameter is performed in the following context: for large values of π the system is close to an integrable system, and the motion in the phase space $(x, \dot{x}, y, \dot{y})$ resembles the surface of a torus[5] (the ergodic trajectories are zero-dimensional,[6] i.e., the order phase is realized; however, for small but finite values of $\pi (\pi < \pi_c = 0.15)$ the motion, just as for $\pi = 0$ (Ref. 1), is stochastic; i.e., the disorder phase is realized.

We shall briefly describe an experiment performed on a computer that was used to calculate the value of π_c. (The first experiments of this type were performed by Contopoulos *et al.*[7]) The computer program was written to solve the equations of

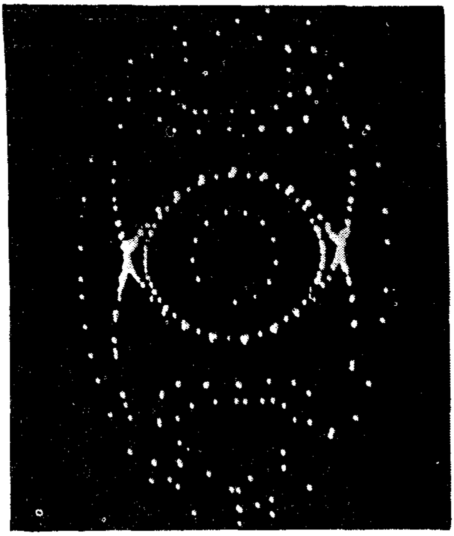

FIG. 1.

motion in (5) for a specified π and to de-excite the points of intersection of the phase trajectory in the phase space $(x, \dot{x}, y, \dot{y})$ with a plane $(y, \dot{y})$ at $\dot{x} > 0$. If the motion is periodic, then the phase trajectory is intersected at a finite number of points, but if it is limited by the surface of the torus, then the points lie on the closed curve in the $(y, \dot{y})$ plane; finally, if the motion is ergodic, then the point wanders chaotically in the $(y, \dot{y})$ plane, covering the finite area densely.

Figure 1 is a picture taken in the $(y, \dot{y})$ plane for $\pi = 4.84$. We see that the points form closed curves, consistent with the KAM theory.[6] The centers of the three, small, closed curves correspond to the periodic, stable trajectories, while the two points of contact on the closed lines correspond to the periodic, unstable trajectories (separatrices[5]). The "macroscopic" regions of the ergodic motion of nonzero dimension have

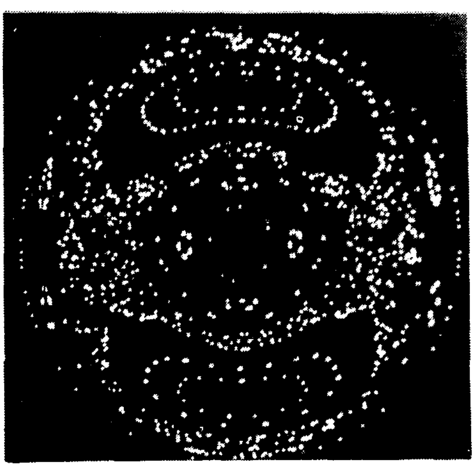

FIG. 2.

FIG. 3.

been observed for the first time in the neighborhood of these trajectories ($\pi = 0.35$ in Fig. 2; compare that with the value in Ref. 7). As the parameter π is decreased, the area traversed by the stochastic motion increases sharply and at a critical value $\pi = \pi_c \approx 0.15$ it becomes almost equal to the entire permissible range of motion in the $(y, \dot{y})$ plane, as illustrated in Fig. 3 (all the points in this figure represent a single trajectory).

The extent to which the effect described above is associated with the quantum theory of phase transitions is not clear[4]; however, it seems that there is a strong correlation between this effect and the phase transitions.

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