Hadron analogs of a classical electron radiation

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The appearance of hadron jets with a large transverse momentum, which flow at nearly a fixed polar angle, is predicted in inelastic hadronic processes at high energies. This effect, which is classical in nature, is attributable to the finite size of nuclear targets.

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One of the most characteristic properties of strong interaction is the limited range of nuclear forces. Hadrons and nuclei are targets whose size is on the order $l \approx m_\pi^{-1} A^{1/3}$, when m_π is the pion mass, and A is the atomic number. Inside such "bags" are quarks and gluons whose properties (spin, type of interaction) are analogous in many ways to those of electrons and photons (if the electric charge is replaced by a color chare).[1] A collision of two hadrons can therefore be reduced to the passage of a quark through a hadronic medium of length l.

We shall consider the emission of a gluon by a quark over a distance $r \lesssim l$ on the basis of the analogy mentioned above. We shall use the well-known solution of the problem of classical radiation of a relativistic electron. To demonstrate the qualitative characteristics of such a process, we shall first analyze the radiation due to scattering of an electron with an energy E by two scattering centers separated from each other by a distance r. The intensity of radiation at a frequency ω in the direction of the vector $\mathbf{n} = \mathbf{k}/\omega$ is given by [2]

$$\frac{dI}{d\omega\, d\Omega} = \frac{e^2}{16\pi^3\omega}\left|\left(\frac{[\mathbf{n}\mathbf{v}_1]}{1-\mathbf{n}\mathbf{v}_1} - \frac{[\mathbf{n}\mathbf{v}_2]}{1-\mathbf{n}\mathbf{v}_2}\right) + \left(\frac{[\mathbf{n}\mathbf{v}_2]}{1-\mathbf{n}\mathbf{v}_2} - \frac{[\mathbf{n}\mathbf{v}_3]}{1-\mathbf{n}\mathbf{v}_3}\right) e^{i\omega r(1-\mathbf{n}\mathbf{v}_2)}\right|^2 \tag{1}$$

where $\mathbf{v}_i$ are the velocities of an electron in the corresponding sections of the path.

In the relativistic case in which the $\mathbf{n}$ and $\mathbf{v}_i$ vectors lie in nearly the same plane and the angles $\theta_i = \arccos(\mathbf{n}\,\mathbf{v}_i/|\mathbf{v}_i|)$ are small but noticeably larger than the characteristic bremsstrahlung angles, i.e., they satisfy the conditions $m/E \ll \theta_i \ll 1$ (m is the electron mass), it follows from Eq. (1) for $|\mathbf{v}_i| \approx 1$ that

$$\frac{dI}{d\omega\, d\Omega} = \frac{e^2}{4\pi^3\omega}\left[(\theta_1^{-1} - \theta_3^{-1})^2 + 4(\theta_2^{-1} - \theta_1^{-1})(\theta_2^{-1} - \theta_3^{-1})\sin^2\frac{\omega r\theta_2^2}{4}\right]. \tag{2}$$

If the distance between the scattering centers varies from 0 to l $(0 \leqslant r \leqslant l)$, then, after averaging Eq. (2) over r, we obtain

$$\frac{dI}{d\omega\, d\Omega} = \frac{e^2}{4\pi^3\omega}\left[(\theta_1^{-1} - \theta_3^{-1})^2 + 2(\theta_2^{-1} - \theta_1^{-1})(\theta_2^{-1} - \theta_3^{-1})\left(1 - \frac{2\sin\frac{\omega l \theta_2^2}{2}}{\omega l \theta_2^2}\right)\right]. \tag{3}$$

The first term characterizes the cone of radiation along the initial and final directions of motion and the second term characterizes the radiation from the region between the scattering centers. This radiation vanishes at $l = 0$, and, as $l \to \infty$, it adds to the first term the radiation from each scatterer along the line that connects them.

If the experiment is set up in such a way that $\theta_2 \ll \theta_1$ and θ_3, then the radiation from the extremities of the path may be ignored, and we see from Eq. (3) that

$$\frac{dI}{d\omega\, d\Omega} = \frac{e^2}{2\pi^3 \omega \theta_2^2}\left(1 - \frac{2\sin\frac{\omega l \theta_2^2}{2}}{\omega l \theta_2^2}\right), \tag{4}$$

i.e., the maximum radiation occurs at an angle

$$\theta_2^{max} = \sqrt{2\pi/\omega l}. \tag{5}$$

This angle is small $(\theta_2^{max} \ll 1)$ at a finite l (say, $\sim m_\pi^{-1} A^{1/3}$) and sufficiently high energies ω, but it is much larger than the angles characteristic of standard bremsstrahlung $(\theta_2^{max} \gg m/E)$, so that the transverse radiation pulse $[p\perp = \omega\theta_2^{max} = (2\pi\omega/t)^{1/2}]$ is large.

This formula can be explained in the following way: the radiation with a large transverse pulse occurs because at a finite length l only those hard components of the radiation have time to form at a relatively large angle for which the formation lengths are small

$$l_f = 2/\omega\theta_2^2 \lesssim l. \tag{6}$$

We note that a characteristic picture of radiation at a sufficiently large angle, which is described by Eq. (4), can be obtained by suppressing the radiation interference at the outer and middle sections of the trajectory. This can be achieved by stipulating that both scattering events occur at large angles (the condition $\theta_2 \ll \theta_1$, θ_3 mentioned above) and that they be accomplished by a direct suppression of radiation at the outer sections. In the picture described above, one scatterer (or both) must therefore be replaced by a semi-infinite waveguide which is oriented parallel by $\mathbf{v}_2$ and which is terminated at the location point of the scatterer.

The equations and results obtained in this case can be applied directly to the interaction of hadrons if we accept the following picture. Let us analyze one of the quarks inside one of the colliding hadrons. Before the collision, this quark is inside the moving "bag," in analogy with the motion of an electron inside a waveguide. During the collision, the coherence of quarks inside the hadron is disturbed ("the electron escapes from the waveguide") and the quark in question may emit on passing through another hadron at the length l those components of its field which initially do not experience confinement (i.e., gluons with large transverse momenta). Subsequently the quark undergoes either an ordinary hadronic collision, deviating from the primary direction or is captured again by the hadron due to its confinement force. Thus the role of confinement reduces to suppression of radiation at the outer limits of the trajectory.[3)]

The emitted gluon becomes a hadron jet. A temporary release from confinement of the hard components of the gluon quark field during the interaction must therefore lead[6] to the appearance of characteristic hadronic jets with large transverse momenta.

Since the interaction of quarks with a gluon differs from electron-photon interaction only by its constant, the axes of the jets, according to Eq. (5), must be pointed at an angle (in the laboratory coordinate system)

$$\theta_J \approx (2\pi m_\pi/\omega A^{1/3})^{1/2} \tag{7}$$

with respect to the collision axis. It can be seen from Eq. (7) that θ_J depends very weakly on the atomic number of the nucleus. We easily estimate from Eq. (4) that the distribution width in the neighborhood of θ_J is small: on the pseudovelocity curve it has the value

$$\Delta(\ln\theta) \approx 0.5. \tag{8}$$

This value is appreciably smaller than the decay width of an isotropic cluster, equal to ~ 2. We recall that the aperture angle of the cone of QCD jets (i.e., the pseudovelocity spread) approaches zero proportionally to the coupling constant $\alpha_s(Q^2)$ as the square of the 4-momentum of the jet is increased.

More energetic jets are emitted at smaller angles in the laboratory coordinate system. If most of the hadrons in the jet depend logarithmically on the energy

$$\bar{n}_J = a\ln(\omega/\omega_0), \tag{9}$$

then the emission angle of the jet decreases exponentially as the number of hadrons in it increases

$$\theta_J = (2\pi m_\pi/\omega_0 A^{1/3})^{1/2} \exp[-\bar{n}_J/2a]. \tag{10}$$

We note that the Cerenkov gluons are emitted in almost the same range of angles if the hadronic medium is characterized by the refractive index n.[4)] The Cerenkov radiation threshold, however, is situated relatively far in energy, since the refractive index n is greater than unity only at very high energies (see Ref. 7 for a detailed discussion of this case. In our case, however, the lower limit on the energy is imposed only by the requirements that 1) a classical analysis by applicable and that 2) the effect of confinement on the gluon be small at the time of its formation.

In summary, a concentration of hadronic jets with large transverse momenta near a fixed polar angle, Eqs. (7) and (10), would favor the classical mechanism of gluon radiation at a finite length in the hadronic medium. This would serve as an experimental verification of the fact that the hadron scattering process at high energies cannot be fully described by the S matrix in the range of $-\infty$ to $+\infty$ with respect to time, and that it requires the incorporation of a finite-time scattering matrix.

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[1)]Below we shall discuss two important differences (gluon self-interaction, and quark and gluon confinement).

[2)]Such formulas are used widely in the study of bremsstrahlung processes in matter[1,2] and of the evolution of the self-field of an electron.[3] The radiation at the end section[4] derived from (1) at $\mathbf{v}_1 = \mathbf{v}_3 = 0$ and, in particular, the systematics of its angular distribution[5] at $\omega r \sim 1$ were also analyzed.

[3)]The second peculiarity—gluon self-interaction—does not affect the results, since we are dealing with classical radiation of gluons by a quark.

[4)]In Eq. (4) θ_2^2 is replaced by $\theta_2^2 - 2\Delta n$, where $\Delta n = n - 1$.

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