ELECTRIC LONGITUDINAL RESONANCE IN SEMIMETALS AND DEGENERATE SEMICONDUCTORS

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When the electron mean free path ℓ is much larger than the plate thickness d and the reflection from the boundaries is specular, the electrons execute a periodic motion between the plate boundaries. The period of this motion is determined by the projection v_z of the electron velocity on the normal to the plane of the plate.

The periodicity of the motion leads to quantization of the z-component of the electron momentum. This quantization is manifest in a number of oscillatory phenomena (both kinetic and in thermodynamic equilibrium). However, since the distances between the quantum energy levels are very small, all these phenomena are observed either in very thin films, or at ultralow temperatures T

$$T(^\circ\mathrm{K})\, d\,(\mathrm{cm}) < 10^{-3}\ .$$

We shall show in this article that in relatively thick plates (but, of course, satisfying the condition $\ell >> d$), there should be observed a classical resonance effect due to the Landau damping [1] and to the existence of extremal values of v_z brought about by Fermi degeneracy.

Assume that a longitudinal alternating electric field of frequency ω and amplitude E_0 ($E_x = E_y = 0$, $E_z = E_0$) is applied to the plate. We are interested in the longitudinal complex admittance of the plate. To calculate it we must solve a system of quasistatic equations (the Boltzmann kinetic equation and the equation of electrostatics). Using the continuity equation, we easily find that in the interior of the plate

$$E_z(z) + \frac{4\pi i}{\omega} j_z(z) = E_0\ , \tag{1}$$

where $j_z(z)$ is the complex current density. Since $j_z(0) = j_z(d) = 0$ (0 and d are the coordinates of the plate boundaries), the field on the two boundaries coincides with the applied field.

We leave out the collision integral from the kinetic equation, assuming that $\omega\tau >> 1$ and $\ell/d >> 1$ ($\tau = \ell/v_F$ and v_F is the Fermi velocity of the electrons). To solve the kinetic equation it is convenient to expand all the functions in terms of $\sin(\pi nz/d)$ ($n = 0, 1, 2, \ldots$) (after first introducing a distribution function f_a which is antisymmetrical in z and vanishes on the plate boundaries; the distribution function that is symmetrical in v_z can be expressed in terms of f_a). Thus, with the aid of Eq. (1), the kinetic equation, and the boundary conditions we obtain the connection between the Fourier components of the current j_n and of the field E_n:

$$j_n = \sigma_n(\omega) E_n\ , \tag{2}$$

where

$$\sigma_n(\omega) = -\frac{i\omega}{4\pi}\left(\frac{d}{\pi n r_D}\right)^2 \left\{1 + \frac{\omega}{2\omega_n}\left[\ln\left|\frac{\omega-\omega_n}{\omega+\omega_n}\right| + i\pi\theta(\omega-\omega_n)\right]\right\}, \tag{3}$$

$$\theta(x) = \begin{cases} 1, & x < 0 \\ 0, & x > 0 \end{cases}, \qquad \omega_n = \frac{\pi v_F}{d} n ,$$

and $r_D = \sqrt{\varepsilon_F / 6\pi N e^2}$ is the Debye-Hückel radius of the electrons, ε_F the Fermi energy, N the electron density, and e the electron charge. In the calculation of (3) we have assumed a quadratic dispersion law (the effects of nonquadratic dispersion will be discussed later on).

The effective dielectric constant $\varepsilon_n = 1 + 4\pi i \sigma_n/\omega$ coincides with classical limit of the Linhard formula [2]. Expressing E_n in terms of E_0 we can calculate the energy absorbed by the plate per unit time:

$$Q(\omega) = \frac{4dE_0^2}{\pi^2} \sum_{k=0}^{\infty} J_{rk+1} ; \qquad J_n = \frac{1}{n^2} \frac{\mathrm{Re}\,\sigma_n}{\left| 1 + \frac{4\pi i}{\omega} \sigma_n \right|^2} . \tag{4}$$

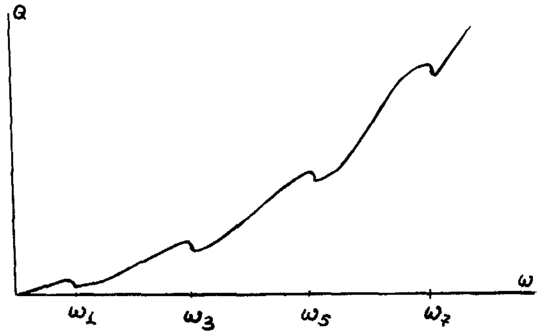
Changing over from summation to integration, we determine the smooth component $\bar{Q}(\omega)$ of the frequency dependence of $Q(\omega)$:

$$\bar{Q}(\omega) \approx \frac{\omega^2 r_D^2}{2\pi v_F} E_0^2 \ln(v_F/\omega r_D), \qquad \omega \ll v_F/r_D . \tag{5}$$

We note that v_F/r_D is of the same order of magnitude as the plasma electron frequency. The limitation $\omega \ll v_F/r_D$ is therefore immaterial even for semimetals and degenerate semiconductors.

On the smooth frequency dependence of the absorbed energy there should be observed singularities near which the "resonant" term in (4) ($\omega \approx \omega_n$) behaves in the following manner (see the figure)

$$Q^n_{\mathrm{res}} \approx \begin{cases} \frac{4n v_F r_D^2}{d^2} E_0^2 \ln^{-2}\left(1 - \frac{\omega}{\omega_n}\right), & \text{for } \omega \lesssim \omega_n , \\ 0, & \text{for } \omega \gtrsim \omega_n . \end{cases} \tag{6}$$



Formula (6) shows that at the resonance points ($\omega = \omega_n$) the derivative $dQ/d\omega$ becomes infinite on the low-frequency side, $\omega \overset{\to}{<} \omega_n$. The weakness of the singularity is due to the fact that it is determined only by the electrons of the limiting point, at which $v_z = v_F$ (see below).

The structure of the electric field $E_z(z)$ and formulas (5) and (6) show that the electrons interact effectively with the longitudinal electric field only in metal layers of thickness r_D adjacent to the plate boundaries. The entire foregoing analysis is therefore valid for good metals with $r_D \lesssim a$ (a is the interatomic distance).

When assessing the possibility of experientally observing the effects considered here, we must compare the derived expressions with expressions and estimates that are valid for dielectrics and not for metals. The effective dielectric constant is equal to $(r_D/d)^2 \ln(v_F/\omega r_D)$.

The characteristic resonant frequency is $\omega_1 = \pi v_F/d$. At $d \sim 0.1$ cm and $v_F \sim 10^7$ cm/sec (semimetals) we have $\omega_1 \sim 10^8$ sec^{-1}. It is necessary here to satisfy the condition $\omega_1 \tau \gg 1$, which imposes stringent requirements on the sample quality.

Dissipative processes smear out somewhat the singularities[1)] at $\omega = \omega_n$ and lead also to an additional energy absorption by the plate $(r_D/\tau)(E_0^2/4\pi)$ at $\omega\tau \gg 1$. This absorption does not depend on the frequency and should serve therefore as a background in the investigation of $Q(\omega)$.

An analysis of the behavior of electrons with an arbitrary dispersion law shows that the shape of the "resonance" curve depends strongly on the form of the Fermi surface. If v_z assumes an extremal value at the saddle point, then a logarithmic dependence is obtained on both sides of ω_n. If v_z reaches an extremal value not at a point but on a line on the Fermi surface, then a square root dependence $Q^n_{res} \sim (\omega - \omega_n)^{1/2}$ should be observed in place of the logarithmic dependence of Q^n_{res} (see (6)). The resonant frequencies are then proportional to v_z^{extr}.

The described longitudinal electric resonance may be useful in investigations of the electron energy spectrum, particularly for a direct measurement of the conduction-electron velocities, and also for investigations of the character of the reflection of the electrons by the plate boundaries. Diffuseness of the scattering, which upsets the periodicity of electron motion, smears out the singularities. A relatively weak magnetic field applied parallel to the plane of the plate may make it possible to vary the angle of encounter of the resonant electrons with the surface (the effect of a magnetic field on the longitudinal resonance calls for a separate study).

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[1] L.D. Landau, Zh. Eksp. Teor. Fiz. 16, 574 (1946).
[2] J. Lindhard. Kgl. Danske. Vidensk. Selsk. Mat-fys. Medd., 28, 8 (1954).

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1) The finite temperature also smears out the singularities. As a rule, however, the collisions play a more important role.