W-boson is produced in the process (3) with a momentum along the motion of the primary proton beam, the muons from its decay (2) will have a transverse-momentum distribution

$$\frac{dN}{dp_\perp} \sim p_\perp \left(\frac{m_W^2}{4} - p_\perp^2\right)^{-1/2} dp_\perp , \qquad (4)$$

i.e., they will have predominantly $p_\perp \approx m_W/2$. The transverse motion of the partons leads to the appearance of a small transverse momentum also for the W boson ($p_{\perp W} \sim \sqrt{2/a}$), which leads to a smearing of the square-root infinity in (4). The energy distributions of the muons from the W boson at fixed observation angles θ will then have narrow peaks near

$$E \approx m_W/2\sin\theta . \qquad (5)$$

Observation of such peaks against the background of the decreasing spectra of the muons from the process (3) will make it possible not only to establish that the W boson is produced, but also to measure its mass on the basis of (5). This conclusion and relation (5) are quite general and do not depend on the model, since they are based on purely kinematic properties of the decay (2). The background from the decays $\pi \to \mu + \nu$ and $K \to \mu + \nu$ is suppressed by the nuclear screening and is separated in experiment from the processes [1]. The figure shows the calculated $d^2\sigma/dEd\Omega$ at muon production angles $\theta = 7$ and 11° (the angle 9° corresponds to the location of the muon-duct axis in the experiment of [1], the aperture of the muon duct is $\Delta\theta = \pm 2°$). It is seem from the figure that the W boson can be observed in the experiment of [1] at a cross section 10^{-37} cm² if its mass is $m_W \lesssim 8$ GeV.

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MOBILITY OF POSITIVE IONS IN SOLID HELIUM

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A vacancion mechanism of ion mobility in a solid is proposed. Two cases of mobility are considered, diffusion and kinetic. The results are compared with the available experimental data.

The mobility of ions in solid helium was first measured by Shal'nikov and co-workers [1]. Shikin [2, 3] proposed a vacancion mechanism of negative-ion mobility. The negative ion (a bubble with a radius of several interatomic distances, in which an electron is contained) moves under the influence of an applied electric field, from the region of higher pressures into a low-pressure region, as a result of the diffusion vacancy fluxes produced in the solid helium. It was also indicated in [3] that no such mechanism can exist for positive-ion mobility, since the probability of a vacancy coming within an interatomic distance of a positive ion is negligibly small, in view of the high pressure produced by polarization around the positive charge. It is known from experiment, however [4], that the mobility decreases exponentially with decreasing temperature and the argument of the exponential is close to that for the vacancy concentration in solid helium (20 - 40°). All the numerical data that follow pertain to helium-4. We propose here a vacancion mechanism for the mobility of positive ions in solid helium.

We consider first a positive ion in solid helium without an external field. The energy of a vacancy situated at a distance r from a positive ion, reckoned from the vacancy energy in the absence of the ion, is determined in the main by the expression

$$V_p(r) = \frac{1}{2}\frac{\alpha \ell^2}{r^4}\omega ,$$

where α is the specific polarizability of helium and ω is the volume of the vacancy (per atom). The vacancy concentration is therefore given by

$$c(r) = c_0 \exp\left\{-\frac{1}{2}\frac{\alpha \ell^2 \omega}{r^4 T}\right\} , \qquad (1)$$

where

$$c_0 = \exp\left\{-\frac{\Delta}{T}\right\} \qquad (2)$$

is the vacancy concentration at infinity.

In formula (2), Δ is the energy needed for a vacancy to appear in solid helium (see [5]). It is seen from (1) that the concentration up to distances determined from the relation

$$\frac{1}{2}\frac{\alpha \ell^2 \omega}{R^4 T} \sim 1 \qquad (3)$$

is much less than c_0, and for $r > R$ it is equal to the concentration c_0 at infinity. Substituting in (3) $\alpha\omega = \omega_0 = 1.96 \times 10^{-25}$ cm^2, $\ell = 4.8 \times 10^{-10}$ cgs esu of charge, and $T \sim 1°$, we obtain $R \sim 10^{-7}$ cm. We note that R depends little on the temperature. The positive ion in the helium is thus surrounded by a sphere of radius $R \sim 10^{-7}$ cm ($\sim$2.8 interatomic distances), inside of which the vacancies do not penetrate. If we now turn on the field, then a force applied to the center of the sphere appears and causes an additional redistribution of the pressures in the helium and with it vacancy fluxes outside the sphere.

Following [3], we can consider two cases: diffusion mobility of the vacancies and the case of delocalized vacancies (vacancions), which apparently takes place in solid helium. In the former case it is necessary to solve the stationary diffusion problem outside our sphere (see [2, 6])

$$\Delta c = 0, \tag{4a}$$

$$c(R) = -\frac{p_n \omega}{T} c_o \tag{4b}$$

Here c stands for the deviation of the concentration from the equilibrium value, and p_n is the pressure on the surface of a sphere of radius R. We obtain this pressure by solving the problem of a δ-force (see [7]) $e\vec{\mathcal{E}}$ applied to the center of the sphere. The vector of the displacement of the medium under the action of the δ-force is

$$\mathbf{u}(\mathbf{r}) = \ell \frac{1+\sigma}{8\pi E(1-\sigma)} \frac{(3-4\sigma)\vec{\mathcal{E}} + \mathbf{n}(\mathbf{n}\vec{\mathcal{E}})}{r} .$$

Here E and σ are the Young's modulus and the Poisson coefficient of the solid helium; $\vec{n} = \vec{r}/r$. Knowing $\vec{u}(\vec{r})$, we obtain the stress tensor σ_{ik} and

$$p_n = \sigma_{ik} n_i n_k = -\frac{(2-\sigma)}{4\pi(1-\sigma)} \frac{e(\vec{\mathcal{E}}\mathbf{n})}{R^2} . \tag{5}$$

We note that the boundary condition (4b) holds only when the requirement $p_n\omega/T \ll 1$ is satisfied. This means that on the surface of a sphere of radius R the vacancy energy due to the deformation resulting from the external force $e\vec{\mathcal{E}}$ is small in comparison with the temperature, whereas the energy due to the polarization around the positive ion $V_p(R)$ is of the order of the temperature. This enables us to regard the motion of a positive ion in solid helium as motion of a vacancion-free sphere with an ion at the center. It is easy to verify that the requirement $p_n\omega T \ll 1$ is satisfied up to fields $\mathcal{E} \sim 10^5$ V/cm. In the estimate we have assumed that $\omega \sim 3.5 \times 10^{-23}$ cm³, $T \sim 1°$, $\sigma \sim 1/3$, and $R \sim 10^{-7}$ cm.

We thus solve Eq. (4a) with the boundary condition (4b) (see [2, 6]) and then calculate the normal velocity of a unit area on the surface of the sphere, using the formula

$$v_n = D \frac{\partial c}{\partial n}\Big|_{r=R} , \tag{6}$$

where D is the coefficient of vacancy diffusion, which in the case of quantum diffusion can be estimated at $D \sim a^2/\tau \sim a^2\varepsilon/\hbar$, where a is the interatomic distance, τ the lifetime of the vacancy on one site, and ε is the width of the vacancion band. We obtain for the ion velocity

$$\left.\begin{aligned} v &= \frac{v_n}{\cos\Theta} = \frac{2-\sigma}{2\pi(1-\sigma)} \frac{a^2\epsilon\omega e\mathcal{E}}{\hbar R^3} \frac{c_o}{T} \\ \mu &= v/\mathcal{E}. \end{aligned}\right\} \tag{7}$$

Here θ is the angle between $\vec{n}$ and $\vec{\mathcal{E}}$, and μ is the mobility.

In the second case the normal velocity per unit area on the surface of the sphere is proportional to the deviation of the vacancion concentration on the sphere surface from the equilibrium value given by (4b), and the average Maxwellian vacancion velocity $\sqrt{2T/M}$ (M is the vacancion effective mass). The exact solution of such a problem (see [3]) leads to the expression

$$v = \frac{v_n}{\cos\theta} = \frac{2-\sigma}{4\pi(1-\sigma)} \frac{e\mathcal{E}\omega c_0}{R^2\sqrt{2\pi MT}}.$$

Recalling c_0, we obtain from (2)

$$\left.\begin{aligned} \mu &= \mu_0(T)e^{-\Delta/T} \\ \mu_0(T) &= \frac{2-\sigma}{2\pi(1-\sigma)} \frac{e\omega}{R^2\sqrt{2\pi MT}} \end{aligned}\right\} . \tag{8}$$

The argument Δ of the exponential is the gap for vacancy production and varies according to [5] approximately from 20 to 35° when the specific volume of the helium varies from 21 to 18 cm^3/mol, which agrees quite well with the change of the argument of the exponential for the mobility (see [4]). To estimate the pre-exponential factor, we calculate M by means of the formula $M \sim (\hbar^2/2a^2)(\varepsilon/z)^{-1}$, where ε is the width of the vacancion band (ε ∿ 2 - 3°, see [5]), a is the interatomic distance (a ∿ 3.6 Å), and Z is the number of nearest neighbors. We obtain $M \sim 2M_{He} \sim 10^{-23}$ g. Substituting the remaining numerical data given above for the pre-exponential factor at T ∿ 1° and $\mu_0 \sim 3$ (cgs esu), which is smaller by approximately two orders of magnitude than the value obtained from the experimental data [4]. The cause of such a discrepancy may be that the quantity actually measured in the experiment is not the mobility but the current, whose temperature depencence should agree qualitatively with the temperature dependence of the mobility, but no exact correspondence can be established between the current and the mobility, owing to the space-charge effect. Thus, the relation (8) agrees qualitatively with experiment [4]. An estimate of the pre-exponential factor by means of (7) leads to approximately the same values as (8).

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GIANT OSCILLATIONS OF SOUND ABSORPTION IN CYLINDRICAL CONDUCTORS UNDER MAGNETIC QUANTIZATION CONDITIONS

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In a preceding paper [1] we have obtained the spectrum of the magnetic surface levels (MSL) in a normal cylinder. We have shown that in a weak