solution of such a problem (see [3]) leads to the expression

$$v = \frac{v_n}{\cos\theta} = \frac{2-\sigma}{4\pi(1-\sigma)} \frac{e\mathcal{E}\omega c_0}{R^2\sqrt{2\pi MT}}.$$

Recalling c_0, we obtain from (2)

$$\left.\begin{aligned} \mu &= \mu_0(T)e^{-\Delta/T} \\ \mu_0(T) &= \frac{2-\sigma}{2\pi(1-\sigma)} \frac{e\omega}{R^2\sqrt{2\pi MT}} \end{aligned}\right\} . \qquad (8)$$

The argument Δ of the exponential is the gap for vacancy production and varies according to [5] approximately from 20 to 35° when the specific volume of the helium varies from 21 to 18 cm^3/mol, which agrees quite well with the change of the argument of the exponential for the mobility (see [4]). To estimate the pre-exponential factor, we calculate M by means of the formula $M \sim (\hbar^2/2a^2)(\varepsilon/z)^{-1}$, where ε is the width of the vacancion band (ε ∿ 2 - 3°, see [5]), a is the interatomic distance (a ∿ 3.6 Å), and Z is the number of nearest neighbors. We obtain $M \sim 2M_{He} \sim 10^{-23}$ g. Substituting the remaining numerical data given above for the pre-exponential factor at T ∿ 1° and $\mu_0 \sim 3$ (cgs esu), which is smaller by approximately two orders of magnitude than the value obtained from the experimental data [4]. The cause of such a discrepancy may be that the quantity actually measured in the experiment is not the mobility but the current, whose temperature depencence should agree qualitatively with the temperature dependence of the mobility, but no exact correspondence can be established between the current and the mobility, owing to the space-charge effect. Thus, the relation (8) agrees qualitatively with experiment [4]. An estimate of the pre-exponential factor by means of (7) leads to approximately the same values as (8).

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[1] K.O. Keshishev, L.P. Mezhov-Deglin, and A.I. Shal'nikov, ZhETF Pis. Red. 12, 234 (1970) [JETP Lett. 12, 160 (1970)].

[2] V.B. Shikin, ibid. 13, 65 (1971) [13, 44 (1971)].

[3] V.B. Shikin, Zh. Eksp. Teor. Fiz. 61, 2053 (1971) [Sov. Phys.-JETP 34, 1095 (1972)].

[4] G.A. Sai-Halasz and A.J. Dahm, Phys. Rev. Lett. 28, 1244 (1972).

[5] V.P. Mineev, Zh. Eksp. Teor. Fiz. 63, 1822 (1972) [Sov. Phys.-JETP 36, No. 5 (1973)].

[6] A.A. Andreev, K. Keshishev, L. Mezhov-Deglin, and A. Shal'nikov, ZhETF Pis. Red. 9, 507 (1969) [JETP Lett. 9, 306 (1969)].

[7] L.D. Landau and E.M. Lifshitz, Teoriya uprugosti (Theory of Elasticity), Nauka, 1965, p. 43 [Addison-Wesley, 1971].

GIANT OSCILLATIONS OF SOUND ABSORPTION IN CYLINDRICAL CONDUCTORS UNDER MAGNETIC QUANTIZATION CONDITIONS

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In a preceding paper [1] we have obtained the spectrum of the magnetic surface levels (MSL) in a normal cylinder. We have shown that in a weak magnetic field with $r_H > R$ (r_H and R are the cyclotron and the cylinder radii, respectively) these levels are responsible for the oscillations of the thermodynamic quantities as functions of the flux, with a period equal to the flux quantum $\phi_0 = hc/e$. Oscillations of this type were named effects of the flux-quantization type [1, 2].

The present paper is devoted to the contribution of the MSL of a cylinder to the sound absorption. It is shown that when longitudinal sound is transmitted through a cylinder, giant absorption oscillations are produced, analogous to the effect of Gurevich, Skobov, and Firsov [3]. Whereas in the bulk metal the period of the oscillations in the absence of transitions between the Landau levels coincides with the de Haas-van Alphen period, in the present case there is a periodicity in the function of the magnetic flux, the distance between peaks being equal to ϕ_0.

Proceeding to the solution of our problem, we write out an expression for the MSL spectrum in a cylinder ($\vec{H} \parallel Oz$) [1]:

$$E_{mn}(p_z) = \epsilon_{mn} + p_z^2/2m^*,$$
$$\epsilon_{mn} = \frac{\hbar^2}{2m^*R^2}\left\{(m+\eta)^2 + m^{4/3}\left[3\pi(n + 3/4)\right]^{2/3}\right\}, \qquad (1)$$

where m* is the electron mass, $\eta = \phi/\phi_0$, and ϕ is the magnetic flux through the cylinder cross section. From the conservation of the energy and of the z-component of the quasimomentum we obtain the condition necessary for the absorption of sound of frequency ω_q, propagating along the z axis (q is the wave vector of the sound):

$$\epsilon_{mn} - \epsilon_{m'n'} + \hbar\omega_q = \frac{p_z}{m^*}\hbar q. \qquad (2)$$

We stipulate that the distance between levels be large, so as to satisfy the condition

$$\epsilon_{mn} - \epsilon_{m'n'} > \hbar q v_F. \qquad (3)$$

Then (2) holds only when n = n' and m = m', and the quasimomentum of the electrons that take part in the absorption is equal to $p_z^{(0)} = m^*s$ (s is the phase velocity of the sound). The condition (3) is equivalent to satisfaction of the inequality $qR < 1$, which correspond to the propagation of elastic waves in a rod, $u = u^{(0)}e^{iqz}$ [4][1]. The coefficient of sound absorption is calculated in analogy with [3], and we present therefore only the final result:

$$\Gamma \cong \frac{1}{4\pi}\Gamma_0 \frac{\lambda}{R} \frac{\Delta\epsilon_m}{kT} \sum_{n,m} \mathrm{ch}^{-2}\left[\frac{\epsilon_{mn}(\eta) - \zeta}{2kT}\right], \qquad (4)$$

where Γ_0 is the sound absorption coefficient of a bulky conductor in a zero field [5], γ is a de Broglie wavelength of the electron, $\Delta\epsilon_m = \epsilon_{m+1,n} - \epsilon_{m,n} \sim \hbar v_F/R$. Expression (4) describes the behavior of the absorption coefficient Γ only near the maxima. A contribution to the absorption is made by electrons

[1] We shall not take into account the possible flexure and torsion waves. The dispersion law of these waves is nonlinear, and in view of the difference between their propagation velocities they can be easily separated in the experiment.

whose energy lies in a narrow interval of the order of kT near the Fermi level ζ. Inside this interval there is produced for the spectrum (1) a set of allowed momentum values $p_z = p_z^{mn}$. When the magnetic field is varied, the positions of the allowed interval changes. When $p_z^{(0)}$ falls in the interval of allowed values at a certain field value, a giant oscillation takes place[2]. A plot of the function $\Gamma(\phi)$ is a series of sharp peaks with a distance $\Delta\phi = \phi_0$ between them.

To determine the absorption coefficient outside the maxima, it is necessary to take into account the electron scattering. This reduces qualitatively to replacement of the δ function that describes the energy conservation law by a smeared (Lorentzian) function with width $\sim 1/\tau$ (τ is the relaxation time). If the conditions $\hbar q^2/2m^* \ll 1/\tau$ and $\omega \ll 1/\tau$ are satisfied, then an investigation of the expression obtained for Γ shows that the giant oscillations are possible if the following inequality is satisfied:

$$\sqrt{\frac{\zeta}{kT}\,\frac{\lambda}{R}} \gg \frac{1}{B} \quad \text{or} \quad q\ell\left(\frac{\lambda}{R}\right)^{1/2} \gg 1, \tag{5}$$

ℓ is the mean free path of the electrons and $B = \sqrt{2kT/m^* q\tau}$. We then obtain the following estimates for the ratio of the largest absorption coefficients to the smallest one:

$$B \sim q\ell\sqrt{\frac{kT}{\zeta}} \gg 1, \quad \frac{\Gamma_{max}}{\Gamma_{min}} \sim q\ell\,\frac{\zeta}{kT}\left(\frac{\lambda}{R}\right)^{3/2} \gg 1,$$

$$B \sim 1, \quad \frac{\Gamma_{max}}{\Gamma_{min}} \sim \left(\frac{\zeta}{kT}\,\frac{\lambda}{R}\right)^{3/2} \gg 1, \tag{6}$$

$$B \ll 1, \quad \frac{\Gamma_{max}}{\Gamma_{min}} \sim (q\ell)^2\left(\frac{\zeta}{kT}\right)^{1/2}\left(\frac{\lambda}{R}\right)^{3/2} \gg 1 \ .$$

As seen from (6), the conditions for experimentally observing the effect are most favorable for metals such as Bi.

It is assumed in the foregoing calculation that the reflection of the "glancing" electrons from the cylinder boundary is highly specular, and that the condition $\hbar v_F/R \gg kT$ is satisfied. It is assumed at the same time that allowance for the weak diffuseness smears out the quantum levels of the "volume" electrons with small values of the magnetic quantum number m. The spectrum corresponding to them, which does not coincide with (1), therefore becomes continuous and makes no contribution to the giant oscillations of the absorption coefficient Γ.

[2]The described effect does not occur for MSL on a plane boundary [6, 7], since the presence in their spectrum of two continuous quantum numbers p_y and p_z (the x axis is directed into the interior of the metal and H is parallel to the surface) does not lead to the formation of p_z-forbidden regions. For high-frequency effects [8], the width of the giant oscillations is determined not only by the temperature but also by the frequency ω. In our case of low-frequency sound we can assume that $\hbar\omega < kT$.

We note in conclusion that the giant oscillations of sound absorption are produced also in the case of a hollow thin-wall cylinder. When the wall thickness is d << R, the electron spectrum takes the form [1, 2]:

$$E_{mn}(p_z) = \frac{\hbar^2}{2m^*}\left\{\frac{(m+\eta)^2}{R^2} + \frac{\pi^2 n^2}{d^2}\right\} + \frac{p_z^2}{2m^*} \quad . \tag{7}$$

The estimates (6) remain in force for the ratio $\Gamma_{max}/\Gamma_{min}$.

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[1] E.N. Bogachek and G.A. Gogadze, Zh. Eksp. Teor. Fiz. 63, 1839 (1972) [Sov. Phys.-JETP 36, No. 5 (1973)].
[2] I.O. Kulik, ZhETF Pis. Red. 11, 407 (1970) [JETP Lett. 11, 275 (1970)].
[3] V.L. Gurevich, V.G. Skobov, and Yu.A. Firsov, Zh. Eksp. Teor. Fiz. 40, 786 (1961) [Sov. Phys.-JETP 13, 552 (1961)].
[4] L.D. Landau and E.M. Lifshitz, Teoriya uprogosti (Theory of Elasticity), Nauka, 1965 [Addison-Wesley, 1971].
[5] A.I. Akhiezer, M.I. Kaganov, and G.Ya. Lyubarskii, Zh. Eksp. Teor. Fiz. 32, 837 (1957) [Sov. Phys.-JETP 5, 685 (1957)].
[6] M.S. Khaikin, ibid. 39, 212 (1960) [12, 152 (1960)]; Usp Fiz. Nauk 96, 409 (1968) [Sov. Phys.-Usp. 11, 785 (1969)].
[7] T.W. Nee and R.E.Prange, Phys. Lett. 25A, 582 (1967).
[8] E.A. Kaner and V.G. Skobov, Advances in Physics, 17, 607 (1968); E.A. Kaner, O.I. Lyubimov, and V.G. Skobov, Zh. Eksp. Teor. Fiz. 58, 730 (1970) [Sov. Phys.-JETP 31, 391 (1970)].

LOW-ENERGY THEOREM FOR THE $\gamma \to 3\pi$ VERTEX AND THE $e^+e^- \to \pi^0\gamma$ PROCESS

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1. One of the interesting results of soft-pion physics is the low-energy theorem recently established by Terent'ev [1] and by Adler et al. [2] for the $\gamma \to 3\pi$ vertex:

$$F_{3\pi} = F_\pi / ef^2 , \tag{1}$$

where $F_{3\pi}$ is an invariant function at the $\gamma \to 3\pi$ vertex in the vero limit of all the arguments, F_π is an analogous function at the $\pi^0 \to 2\gamma$ vertex in the limit as $\mu \to 0$, μ is the pion mass, and f = 83 MeV. An experimental confirmation of relation (1) when F_π is replaced by the experimental value of the $\pi^0 \to 2\gamma$ decay constant, i.e., when $F_\pi(0) = F_\pi(\mu^2)$, would mean the existence of a certain internal mechanism that makes this decay dynamically not forbidden, in contradiction to the prediction of the naive PCAC theory. Experiments aimed at determinating $F_{3\pi}$ were proposed in various papers [1, 3]. It is clear, however,