We note in conclusion that the giant oscillations of sound absorption are produced also in the case of a hollow thin-wall cylinder. When the wall thickness is d << R, the electron spectrum takes the form [1, 2]:

$$E_{mn}(p_z) = \frac{\hbar^2}{2m^*}\left\{\frac{(m+\eta)^2}{R^2} + \frac{\pi^2 n^2}{d^2}\right\} + \frac{p_z^2}{2m^*} . \tag{7}$$

The estimates (6) remain in force for the ratio $\Gamma_{max}/\Gamma_{min}$.

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LOW-ENERGY THEOREM FOR THE $\gamma \to 3\pi$ VERTEX AND THE $e^+e^- \to \pi^0\gamma$ PROCESS

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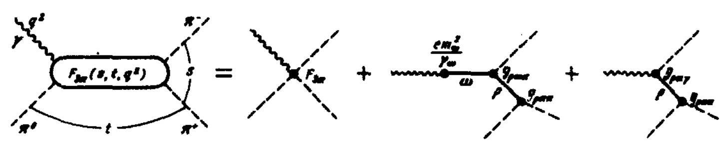
1. One of the interesting results of soft-pion physics is the low-energy theorem recently established by Terent'ev [1] and by Adler et al. [2] for the $\gamma \to 3\pi$ vertex:

$$F_{3\pi} = F_\pi / ef^2 , \tag{1}$$

where $F_{3\pi}$ is an invariant function at the $\gamma \to 3\pi$ vertex in the vero limit of all the arguments, F_π is an analogous function at the $\pi^0 \to 2\gamma$ vertex in the limit as $\mu \to 0$, μ is the pion mass, and f = 83 MeV. An experimental confirmation of relation (1) when F_π is replaced by the experimental value of the $\pi^0 \to 2\gamma$ decay constant, i.e., when $F_\pi(0) = F_\pi(\mu^2)$, would mean the existence of a certain internal mechanism that makes this decay dynamically not forbidden, in contradiction to the prediction of the naive PCAC theory. Experiments aimed at determinating $F_{3\pi}$ were proposed in various papers [1, 3]. It is clear, however,



that what is measured in experiment is not $F_{3\pi}$ but $|F_{3\pi}(s, t, q^2)|^2$, and the situation is made more complicated by the fact that even near the threshold of these reactions a noticeable contribution is made by the corrections connected with the exchange of certain mesons, and in order to obtain the values of $F_{3\pi}$ from these data it is necessary to know the phase shifts of the interference terms with the resonant contributions. It turns out that these phase shifts can be determined for the $e^+e^- \to \pi^0\gamma$ process. We write down the function $F_{3\pi}(s, t, q^2)$ in the form (the notation is indicated in the figure)

$$F_{3\pi}(s,t,q^2) = F_{3\pi} + \frac{2g_{\rho\pi\gamma}\, g_{\rho\pi\pi}}{m_\rho^3}\left[\frac{s}{m_\rho^2 - s} + (s \to u) + (s \to t)\right] + \qquad (2)$$

$$+ \frac{2eg_{\rho\pi\pi}\, g_{\rho\omega\pi}}{\gamma_\omega m_\omega}\, \frac{q^2}{m_\omega^2 - q^2}\left[\frac{1}{m_\rho^2 - s} + (s \to u) + (s \to t)\right],$$

where the constants g and γ_V are connected with the corresponding partial widths in the following manner (see [4] concerning all the experimental data):

$$\Gamma_{\rho\pi\pi} = \frac{1}{3}\,\frac{g_{\rho\pi\pi}^2}{16\pi}\left(1 - \frac{4\mu^2}{m_\rho^2}\right)^{3/2} \qquad (3a)$$

$$\Gamma_{\omega\to 3\pi} = (m_\omega - 3\mu)^4 (m_\rho^2 - 4\mu^2)^{-2} (\mu^2/m_\omega)(g_{\rho\pi\pi}^2/16\pi)(g_{\rho\omega\pi}^2/4\pi)\, W(m_\omega), \qquad (3b)$$

$$W(m_\omega = 787\ \mathrm{meV}) = 3.56$$

$$\Gamma_V = \frac{\alpha^2}{3}\,\frac{4\pi}{\gamma_V^2}\, m_V, \qquad (3c)$$

$$\Gamma_{V\to\pi\gamma} = \frac{g_{V\pi\gamma}^2}{96\pi}\, m_V (1 - \mu^2/m_V^2)^3, \qquad (3d)$$

where $V = \rho, \omega, \phi$. The GSW model [5] was used for the $\omega \to 3\pi$ decay. The contribution of the ϕ meson will not be taken into account because of the small coupling constants. It is clear that we can neglect the imaginary parts of g and γ_V near the threshold of the reactions[1)], so that we need concern ourselves with the signs of the interference terms. Under the VDM assumption, we have $g_{\omega\pi\gamma} = eg_{\rho\omega\pi}/\gamma_\rho$ for the $\omega \to \pi^0\gamma$ decay, which is well satisfied for the contemporary experimental data ($g_{\rho\omega\pi}$ from (3b)). From SU(3) symmetry we have

$$g_{\rho\pi\gamma} = \frac{1}{3}\, g^D, \qquad g_{\omega\pi\gamma} = \frac{1}{3}\, g^D + \frac{1}{\sqrt{3}}\, g^S, \qquad (4)$$

$$g_{\phi\pi\gamma} = \frac{1}{3}\, g^D - \frac{1}{\sqrt{6}}\, g^S.$$

[1)] We note that only the two-pion intermediate state need be taken into account in the calculation of Im $g_{\rho\to\pi\gamma}$, and if (1) is satisfied we get Im $g_{\rho\to\pi\gamma} = -g_{\rho\pi\pi}F_{3\pi}/96\pi\,(m_\rho^2 - 4\mu^2)^{3/2}$, from which we obtain $\Gamma_{\rho\to\pi\gamma} \gtrsim 12$ keV as the lower limit of the $\rho \to \pi\gamma$ decay.

For the $\omega\phi$ mixing we chose the value $\cos\theta = \sqrt{2/3}$. Although even Eq. (4) is not satisfied quite satisfactorily at the present time, we can nevertheless conclude quite definitely that g^D and g^S have the same sign, so that $g_{\omega\pi\gamma}g_{\rho\pi\gamma} > 0$. From SU(3) symmetry we have also the relation $\text{sign}\gamma_\omega = \text{sign}\gamma_\rho$, so that all the interference terms have the same sign, $\eta_{3\pi} = \text{sign}[g_{\rho\pi\gamma}g_{\rho\pi\pi}/F_{3\pi}]$.

We consider now the process $e^+e^- \to "\gamma" \to \pi^0\gamma$. We write the virtual photon $\to\pi^0$ + photon vertex in the form

$$F_\pi(q^2) = F_\pi + \frac{eg_{\rho\pi\gamma}}{\gamma_\rho m_\rho}\frac{q^2}{m_\rho^2 - q^2} + (\rho \to \omega). \tag{5}$$

The cross section of the process is expressed in terms of $F_\pi(q^2)$ in the following manner:

$$\sigma_{e^+e^- \to \pi^0\gamma} = \frac{\alpha}{24}|F_\pi(q^2)|^2\left(1 - \frac{\mu^2}{q^2}\right)^3. \tag{6}$$

The interference terms F_π with the resonant contribution have the signs of the quantities $eg_{\rho\pi\gamma}/\gamma_V F_\pi$ ($V = \rho, \omega$). It is easily seen, however, that

$$\eta_{3\pi} = \eta = \text{sign}\,\frac{eg_{V\pi\gamma}}{\gamma_V F_\pi}. \tag{7}$$

Thus, the form factor of the pion in the reaction $e^+e^- \to "\gamma" \to \pi^+\pi^-$ at low energies is described by the formula

$$F(q^2) = 1 - \frac{g_{\rho\pi\pi}}{\gamma_\rho} + \frac{g_{\rho\pi\pi}}{\gamma_\rho}\frac{1}{1 - q^2/m_\rho^2},\ q^2 < 1\ \text{GeV}^2$$

with $(g_{\rho\pi\pi}/\gamma_\rho) > 0$. If (1) holds now, then we obtain (7). The cross section (6) is very sensitive to η in the region $q^2 \sim 0.3 - 0.4\ \text{GeV}^2$. The table lists the values of $\sigma(e^+e^- \to \pi^0\gamma)$ for different values of q^2 and $\eta = \pm 1$. For $\Delta_V = |eg_{V\pi\gamma}/\gamma_V F_\pi m_V|$ we have used the values $\Delta_\rho = 1.15$ and $\Delta_\omega = 0.7$.

$\sigma(e^+e^- \to \pi^0\gamma) \times 10^{33}\ \text{cm}^2$

q^2, Gev^2 / η	0.3	0.35	0.4	0.45
+ 1	0.54	0.8	1.16	2.3
− 1	0.046	0.16	0.35	1.07

Such cross sections can already be measured at the present time.

2. The presence of the foregoing possibility of determining the sign of the interference terms makes it preferable to determine $F_{3\pi}$ by direct experiment in the reaction $e^+e^- \to "\gamma" \to 3\pi$. Although this cross section near the threshold is only 10^{-35} cm^2 in the most favorable case, this difficulty will be resolved in time. Since we are working near the threshold, when $q^2 \sim (3 - 4\mu)^2$, we can confine ourselves in (2) to the first approximation in q^2/m_V^2 ($V = \rho, \omega$). The expression for the cross section is then

$$\sigma_{e^+e^-\to 3\pi} = \frac{a}{9(2\pi)^2}\frac{\mu^2}{2^8W^3}(W-\mu)(W-3\mu)^4[1+G(W)]\,|F(W)|^2$$

$$G(W)\Big|_{W=3\mu} = 12$$

$$F(W) = F_{3\pi} + \frac{6eg_{\rho\pi\pi}g_{\rho\omega\pi}}{\gamma_\omega m_\rho^2 m_\omega}\frac{q^2}{m_\omega^2 - q^2} + \qquad (8)$$

$$+\frac{3(W\mu+\mu^2)}{m_\rho^2}\left[\frac{2g_{\rho\pi\gamma}g_{\rho\pi\pi}}{m_\rho^3} + \frac{2e\,g_{\rho\omega\pi}g_{\rho\pi\pi}}{\gamma_\omega m_\omega m_\rho^2}\frac{q^2}{m_\omega^2-q^2}\right],$$

where $W = \sqrt{q^2}$ and the approximations in phase space in the last integration with respect to E (which is the energy, say, of the π^+ meson) are $E^2 - \mu^2 \simeq 2\mu(E-\mu)$ and $W^2 - 2WE + \mu^2 \simeq 4\mu^2$.

We present the values of $\sigma(e^+e^- \to 3\pi)$ at the point $W = 4\mu$ for the cases[2] $\eta = \pm 1$ and $F_{3\pi} = 0$.

$$\sigma^{(+)}_{e^+e^-\to\pi^0\gamma} \approx 0.87\cdot 10^{-35}\ \text{cm}^2,$$

$$\sigma^{(-)}_{e^+e^-\to\pi^0\gamma} \approx 0.85\cdot 10^{-36}\ \text{cm}^2.$$

At $F_{3\pi} = 0$

$$\sigma_{e^+e^-\to\pi^0\gamma} \approx 3.6\cdot 10^{-36}\ \text{cm}^2.$$

Thus, two independent measurements of the total cross section of the process $e^+e^- \to \pi^0\gamma$ at $q^2 \sim 0.3 - 0.4\ \text{GeV}^2$ and of the total cross section of $e^+e^- \to 3\pi$ near the threshold give a perfectly defined answer for $F_{3\pi}$.

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SLOPE OF CONE IN pp SCATTERING AND THE "QUASIPOLE" MODEL

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Recent measurements of the cross sections for pp scattering [1 - 3] in the CERN colliding beams at $s \sim 500 - 3000\ \text{GeV}^2$ and their comparison with data

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[2]) We note that if only $F_{3\pi}$ is taken into account in (8) we obtain for $\sigma(e^+e^- \to 3\pi)$ at $W = 4\mu$ the value $\sim 0.9 \times 10^{-36}\ \text{cm}^2$, which is smaller by one order of magnitude than the result of [7]. It seems to us, however, that a factor 1 was left out from formula (A.20) of [7].