$$\sigma_{e^+e^-\to 3\pi} = \frac{\alpha}{9(2\pi)^2}\frac{\mu^2}{2^8 W^3}(W-\mu)(W-3\mu)^4[1+G(W)]\,|F(W)|^2$$

$$G(W)\Big|_{W=3\mu} = 12$$

$$F(W) = F_{3\pi} + \frac{6eg_{\rho\pi\pi}g_{\rho\omega\pi}}{\gamma_\omega m_\rho^2 m_\omega}\frac{q^2}{m_\omega^2 - q^2} + \tag{8}$$

$$+\frac{3(W\mu+\mu^2)}{m_\rho^2}\left[\frac{2g_{\rho\pi\gamma}g_{\rho\pi\pi}}{m_\rho^3} + \frac{2e\,g_{\rho\omega\pi}g_{\rho\pi\pi}}{\gamma_\omega m_\omega m_\rho^2}\frac{q^2}{m_\omega^2-q^2}\right],$$

where $W = \sqrt{q^2}$ and the approximations in phase space in the last integration with respect to E (which is the energy, say, of the π^+ meson) are $E^2 - \mu^2 \simeq 2\mu(E-\mu)$ and $W^2 - 2WE + \mu^2 \simeq 4\mu^2$.

We present the values of $\sigma(e^+e^- \to 3\pi)$ at the point $W = 4\mu$ for the cases²) $\eta = \pm 1$ and $F_{3\pi} = 0$.

$$\sigma^{(+)}_{e^+e^-\to\pi^0\gamma} \approx 0.87\cdot 10^{-35}\ \mathrm{cm}^2,$$

$$\sigma^{(-)}_{e^+e^-\to\pi^0\gamma} \approx 0.85\cdot 10^{-36}\ \mathrm{cm}^2.$$

At $F_{3\pi} = 0$

$$\sigma_{e^+e^-\to\pi^0\gamma} \approx 3.6\cdot 10^{-36}\ \mathrm{cm}^2.$$

Thus, two independent measurements of the total cross section of the process $e^+e^- \to \pi^0\gamma$ at $q^2 \sim 0.3 - 0.4$ GeV² and of the total cross section of $e^+e^- \to 3\pi$ near the threshold give a perfectly defined answer for $F_{3\pi}$.

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SLOPE OF CONE IN pp SCATTERING AND THE "QUASIPOLE" MODEL

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Recent measurements of the cross sections for pp scattering [1 - 3] in the CERN colliding beams at $s \sim 500 - 3000$ GeV² and their comparison with data

²) We note that if only $F_{3\pi}$ is taken into account in (8) we obtain for $\sigma(e^+e^- \to 3\pi)$ at $W = 4\mu$ the value $\sim 0.9 \times 10^{-36}$ cm², which is smaller by one order of magnitude than the result of [7]. It seems to us, however, that a factor 1 was left out from formula (A.20) of [7].

obtained earlier [4] with the accelerator of the Institute of High Energy Physics (IHEP) have revealed two important circumstances:

a) The diffraction cone slope parameter b decreases by ~ 1.5 GeV^{-2} on going from the region of small $|t| \lesssim 0.15$ GeV^2 to the region of large $|t| > 0.15$ GeV^2.

b) b is not a strictly linear function of ln s and varies more slowly.

These two properties of b do not agree with the traditional notion that the predominant role in the elastic-scattering amplitude in the case of large s and small t is played by a Pomeranchuk pole with $\alpha_p(t) = 1 + \alpha' t$ (this would mean $b = b_0 + 2\alpha' \ln s$, where b_0 and α' are constants).

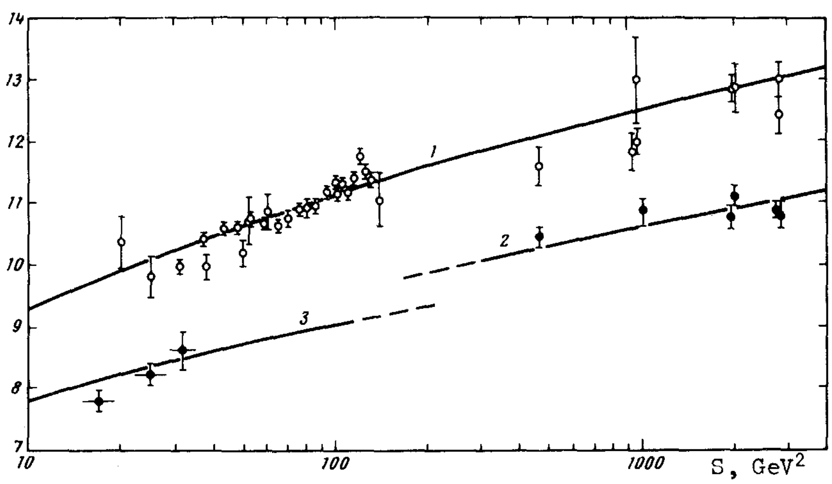
We shall show in this article that the foregoing pecularities of b can be explained with the aid of the model proposed by us in our preceding paper [5]. In this model the diffraction-cone parameter is given by

$$b \equiv \frac{d}{dt}\left(\ln \frac{d\sigma}{dt}\right) = b_0 + c\sqrt{\ln s/(t_0 - t)}\,, \qquad (1)$$

where b_0 and c are constants. It is easy to see that expression (1) has precisely the required properties: the dependence on ln s is slower than linear, and the t-dependence is such that b decreases with increasing t.

The figure shows a comparison of the predictions of the model with the experimental data. The parameters $b_0 = 4.98$ and $c = 1.23$ were obtained by fitting (1) to the largest group of experimental data, corresponding to measurements [2, 3, 6] at small t (the mean value of t for this group is $|\bar{t}| = 0.69$ GeV^2). In this case $\chi^2 = 75$. For comparison we show that fitting of the same data with the aid of a dependence linear in ln s (corresponding to a Pomeron exchange) results in $\chi^2 = 91$.

The obtained values of b_0 and c were used to calculate the curves at $|\bar{t}| = 0.09$, and also at the values $|\bar{t}| = 0.22$ and 0.32 corresponding to the measurements made at large t in [1, 2] and [7], respectively. We see that the theoretical curves describe well all three groups of the experimental data. The dependence of b on $\sqrt{\ln s}$ makes it possible to reconcile the data at $s \sim 10^3$ GeV^2 and $s < 140$ GeV^2 without any "shifting" of the latter, as was proposed in [3].



Parameter of the diffraction cone slope in the "quasipole" model. Curves 1, 2, and 3 were calculated from formula (1) with $b_0 = 4.98$ and $c = 1.23$ for the values $|\bar{t}| = 0.09$, 0.22, and 0.32 GeV^2, corresponding to the average momentum transfers at which the measurements were performed in [2, 3, 6] ( ), [1, 2] ( ), and [7] ( ), respectively.

We now explain briefly the physical mechanism that leads to the dependence (1). As shown in [5], an important role should be played in the asymptotic form of the scattering amplitude by the widths of the resonances exchanged in the t-channel (these widths are usually ignored in the derivation of the asymptotic formulas [8]). Allowance for the width leads to the existence of Regge poles on the physical sheet of the complex J plane only at $t > 0$. When $t < 0$, the poles go off through the fixed cut from $-\infty$ to $J = \alpha(0)$ to an unphysical sheet, where they form complex-conjugate pairs with the other poles. At small t and not very large s, these poles lie close to the real axis and determine the contribution from the cut, thus imitating the presence of ordinary moving poles.

Thus, in spite of the fact that the only amplitude singularities at $t \leq 0$ are branch points, the amplitude behaves in a definite region of s and t almost exactly as if it had ordinary Regge poles. (In this sense we can call this a "quasipole model".) With increasing s and t, the deviation from the pole model becomes stronger. At $t^2 \ln s < 1$ the amplitude in such a model is given by

$$T^{\pm}(s,t) = \beta(t)\begin{pmatrix}-1\\ i\end{pmatrix} e^{-\frac{i\pi\alpha_0}{2}} s^{\alpha_0} \exp\left\{-2\gamma\sqrt{\frac{\alpha'}{\pi}(t_0 - t)(\ln s - \frac{i\pi}{2})}\right\}, \tag{2}$$

where γ is a parameter connected with the width of the resonances, α' is the slope of the trajectory at $t > 0$, and t_0 is the lowest thershold in the t-channel and equals $4\mu_\pi^2$ in this case.

We assume now that at $t > 0$ there exists a family of resonances lying on the leading vacuum trajectory with $\alpha(0) = 1$. Then the model indicated above can be used to describe diffraction scattering. This means that at $t \leq 0$ the asymptotic form of the amplitude is determined not by a pole but by a fixed cut with $\alpha_c = (0) = 1$. Then $s_{tot} \sim (\ln s)^{-1/2}$ as $s \to \infty$. If we interpret, as usual, the experimental data in terms of the Regge trajectories, then we can, starting with (2), introduce an "effective" vacuum trajectory

$$\alpha_{eff} \approx 1 - c\sqrt{(t_0 - t)/\ln s}\,, \tag{3}$$

where $c \equiv 2\gamma\sqrt{\alpha'\pi}$. We see that α_{eff} has a curvature $d\alpha_{eff}/dt \sim (t_0 - t)^{-1/2}$ (see also [9]). From this follows directly expression (1) for the slope parameter b.

We note in conclusion that the quantity c obtained by the fitting is somewhat larger than that obtained from estimates based on typical values of the meson-resonance widths. It must be recognized, however, that (2) was derived by making a number of simplifying assumptions. It was assumed, in particular, that the reduced width γ is constant, whereas the threshold behavior in the t-channel necessitates allowance for a dependence of γ on t [5].

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GEOMETRIC RESONANCE IN ELECTROMAGNETIC EXCITATION OF SOUND IN METALS

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The amplitude of excited sound in a strong magnetic field parallel to the surface experiences oscillations corresponding to geometric resonance. The oscillation amplitude is proportional to the deformation-potential tensor at a definite point of the Fermi surface.

The interaction of electrons with sound is usually described with the aid of the deformation tensor $\lambda_{ik}(\vec{p}) = \lambda_{ik}(-\vec{p})$, which characterizes the change of the dispersion law $\varepsilon(\vec{p})$ upon deformation. The corresponding volume force in the equation of motion of the elastic medium is

$$F_i = \frac{\partial}{\partial x_k} < \lambda_{ik} f > \tag{1}$$

($f\delta(\varepsilon - \mu)$ is the non-equilibrium part of the distribution function, and the angle brackets denote integration over the Fermi surface). When the electron system is perturbed by an external electromagnetic wave, the force (1) is responsible for the deformation mechanism of sound excitation.

It is of interest to look into the singularities of sound excitation under anomalous skin effect conditions. It is known that under such conditions the contribution of different electron groups to the penetration of the electromagnetic field into the interior of the sample is quite different, so that the form of $\varepsilon(\vec{p})$ can be reconstructed from the experimental data. We show below that when sound is excited in the presence of a strong magnetic field H_0 under conditions when the radius R of the Larmor orbit exceeds the length of the sound wave, there should be observed sound-amplitude oscillations of appreciable magnitude, due to the electrons from the extremal Fermi-surface section perpendicular to H_0, which glide parallel to the sample surface. This makes it possible to determine the value of $\hat{\lambda}(\vec{p})$ on the Fermi surface directly from experimental data on sound generation.

We consider the excitation of sound in a half-space $z > 0$ in the presence of a strong magnetic field H_0 parallel to the surface ($\gamma = (\Omega\tau)^{-1} << 1$, Ω is the cyclotron frequency, and τ is the relaxation time). Let us find the amplitude of the transverse sound wave excited by the force (1). As usual in the anomalous skin effect, we use in (1) the distribution function f without allowance for the boundary conditions, we neglect the field E_z, and continue the field $E_{x,y}$ in even fashion to the region $z < 0$. It is easy to show that far from the surface

$$u_i = \frac{\pi e}{i\rho s^2} < \frac{\lambda_{iz}}{\Omega} \int_{-\infty}^{\phi} d\phi' \, \mathrm{v}\, \mathrm{E}(k) \exp \int_{\phi}^{\phi'} \gamma \, d\phi'' \sin \frac{k}{\Omega} \int_{\phi}^{\phi'} v_z d\phi'' > ,$$