Multielectron bubbles in liquid helium

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The equilibrium characteristics of a multielectron bubble in liquid helium are calculated. Various mechanisms of the instability of such a bubble are investigated. The results of this investigation lead to the conclusion that the multielectron bubble in liquid helium is stable.

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A recent experiment[1] has revealed the onset of multielectron bubbles that move into the interior of the liquid phase, when the charged surface of liquid helium looses its stability. The purpose of the present article is to describe the simplest properties of multielectron bubbles.

The bubble dimension R is determined from the condition that the total energy W of the complex made up of z electrons plus the deformation of the liquid helium be a minimum, where

$$W = W_C + W_\perp + W_\sigma,\; W_C = 2^{-1} z^2 e^2 R^{-1},\quad W_\perp = 2^{-1}\gamma \hbar^2 m^{-1}\delta^{-2},$$

$$\delta^3 = 2\gamma a_0 R^2 z^{-1},\quad W_\sigma = 4\pi R^2 \sigma, \tag{1}$$

W_C is the Coulomb-repulsion energy, z is the number of electrons in the bubble, $W_\perp$ is the energy of localization of the electrons near the helium surface in the radial direction (the value δ in this part of the energy is obtained by minimizing with respect to δ the sum $ze^2(R-\delta)^{-1}+\gamma\hbar^2 m^{-1}\delta^{-2}$, $\gamma\approx 1$, whence $\delta^3=2\gamma a_0 R^2 z^{-1}, a_0=(h^2/me^2)$ is the Bohr radius; taking into account the numerical value of the Bohr radius $a_0\approx 0.5$ Å in the expected order of magnitude $R\gg 10^{-7}$ cm, we can conclude that $\delta\ll R$), W_σ is the energy due to the appearance of the additional liquid-helium surface, and σ is the surface-tension coefficient on the vapor-liquid boundary of the liquid helium. The possible entropy contribution to (1) can be disregarded, if it is assumed that the temperature of the medium is small enough. The Fermi energy of the electrons located on a sphere of radius R is of higher order of smallness in the parameter $a_0 R^{-1}$ compared with $W_\perp$.

Minimizing (1) with respect to R, we obtain the value of the equilibrium radius

$$R^3 = R_C^3(1+\Delta),\qquad R_C^3 = \frac{z^2 e^2}{16\pi\sigma},\quad \Delta = \left(\frac{\gamma a_0}{4R_C}\right)^{1/3} << 1. \tag{2}$$

It is obvious that allowance for $W_\perp$ in the total expression (1) for W leads to a small correction $\sim\Delta$ in the radius R. However, as will be shown below, calculation of R with accuracy higher than R_C is essential to assess the deformation stability of a multielectron bubble.

Proceeding to a discussion of the stability of such a formation, it must be noted first that the spherically symmetrical solution (2) is valid if the gravitational energy of the bubble, which tends to flatten the bubble, is lower than the surface-tension energy, which is minimal when the bubble is spherical. The explicit form of the condition for gravitational stability is (ρ is the density of the helium and g is the acceleration due to gravity)

$$R^2 \lesssim R_g^2 \qquad R_g^2 \approx 3\sigma\rho^{-1}g^{-1}. \tag{3}$$

For the parameters ρ and σ that are characteristic of liquid helium we have $R_g \approx 10^{-3}$ cm. Taking the definition (2) of R into account, we can conclude that sufficiently stable bubbles with $R < R_g$ can have a total charge z not larger than $z \approx 10^5$. Thus, in the absence of an external pressure to stabilize additionally the many-electron production, the surface density $n_S = z/4\pi R^2$ in the bubble has a scale comparable with the densities $n_S \approx 10^{10}$ cm^{-2} obtainable on a flat surface of liquid helium in the region of its stability loss.

The next source of instability is tunneling decay of the bubble. Calculation of the characteristic bubble tunneling-decay time τ is analogous to the problem of α decay in nuclear physics[2]

$$\tau^{-1} \approx \omega_0 \exp\left(-\frac{2}{\hbar} S\right), \qquad \omega_0 \approx \hbar m^{-1}\delta^{-2}, \tag{4}$$

$$S = \frac{2}{3} z e^2 \left(\frac{2m}{V_C}\right)^{1/2} \left(\frac{V_0}{V_C}\right)^3, \quad V_C = \frac{ze^2}{2R}, \quad \frac{V_0}{V_C} << 1 \tag{5}$$

m is the mass of the free electron, V_0 is the potential barrier to the penetration of the electron from the vacuum into liquid helium, and $V_0 \approx 1$ eV. Taking the dependence of V_C on z into account, it is easy to establish that $S \propto z^{-1/6}$, i.e., the tunneling probability increases with increasing z. However, even for the maximal values $z \approx 10^5$ corresponding to bubbles with $R \lesssim 10^{-3}$ cm, S is still quite large: $S\hbar^{-1} \approx 10^2$. At these values of the argument of the exponential in (4), the probability of tunneling decay of large bubbles is negligibly small. It should be noted that the inequality $V_0/V_C \ll 1$, which is essentially used to simplify the general expression (5) for S, holds true up to $z \gtrsim 10$. On the other hand, in the region $z \leqslant 10$ tunneling decay of a multielectron bubble becomes impossible.

The third type of instability (arbitrarily called by us deformation instability) is the result of competition between the Coulomb forces which tend to stretch the spherical bubble into an elipsoid, and the surface-tension forces that maintain the bubble in the spherical state. By way of a criterion of the deformation instability we can use the relation that results from the solution of the problem of deformation instability of a liquid drop with a charged surface[3]

$$16\pi R^3\sigma > z^2 e^2 \quad \text{or} \quad R^3 > R_C^3 . \tag{6}$$

The necessary conditions for the use of inequality (6) are high conductivity of the surface of the drop and its incompressibility i.e., the invariance of the drop volume under weak elliptic deformation. Both conditions are satisfied with good accuracy for multielectron bubbles; the former by virtue of the high mobility of the electrons over the liquid-helium surface (an experimental fact), and the latter because bubble deformation without change of volume is much more probable than with a change (the compressibility of the bubble is quite small). Taking the foregoing remarks into account and substituting in (6) expression (2) for R, we arrive at the requirement

$$\Delta > 0. \tag{6a}$$

Thus, the condition of deformation instability is satisfied (to the extent that $\Delta > 0$), but in contrast to the gravitational instability, which is inessential for bubbles with $R < R_g$, and the tunneling instability, the presence of which can be neglected, the inequality (6a) turns out to be weakly pronounced. In view of the smallness of Δ, the question of the deformation instability of multielectron bubbles seems to call for a more thorough study.

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