Gluon condensate and leptonic decays of vector mesons

A. I. Vaĭnshteĭn, V. I. Zakharov, and M. A. Shifman

Institute of Theoretical and Experimental Physics
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We consider the power-law corrections to the asymptotic-freedom predictions for vacuum polarization by a photon. The corrections are determined by the square of the gluon-field stress tensor (gluon condensate) averaged over the vacuum. The analysis makes it possible to calculate the lepton widths of the ρ^0 and ϕ^0 mesons. The results agree well with experiment.

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We calculate in this article the constants of the leptonic decays of ρ and ϕ mesons:

$$g_\rho^2/4\pi \approx \frac{2\pi}{e} \approx 2.3; \tag{1}$$

$$\frac{g_\phi^2}{g_\rho^2} \approx \frac{9}{2}\,\frac{m_\phi^2}{m_\rho^2 - 4\pi^2\left(f_K^2\dfrac{m_K^2}{m_\rho^2} - f_\pi^2\dfrac{m_\pi^2}{m_\rho^2}\right)}\,e^{1-m_\phi^2/m_\rho^2} \approx 10\,. \tag{2}$$

Here e is the base of the natural logarithm, $m_\phi, m_\rho, m_K, m_\pi$ are the masses of the corresponding mesons, $f_K f_\pi$ are the constants of the decays $K\to\mu\nu$ and $\pi\to\mu\nu$ ($f_K \approx 1.25 f_\pi$; $f_\pi \approx 0.95 m_\pi$). The constants g_ρ and g_ϕ are defined in standard fashion in such a way that the width of the electronic decay is equal to $\Gamma(V\to e^+e^-)=(4\pi/3)\alpha^2(m_V/g^2{}_V)$. The predictions (1) and (2) agree with experiment within the limits of the experimental errors.

The derivation of relations (1) and (2) is connected with consideration of certain principal questions of quantum chromodynamics. A central assumption is the nonvanishing of the vacuum mean value

$$<0\,|\,G^a_{\mu\nu}G^a_{\mu\nu}|\,0>, \tag{3}$$

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where $G^a_{\mu\nu}$ is the gluon-field strength tensor.

To explain how the vacuum mean value (3) enters into consideration, we recall first the standard method of operation in quantum chromodynamics (see, e.g.,[1]).

It follows from asymptotic freedom that, say, the polarization of a hadron vacuum in the deep-Euclidean region is reliably calculated in perturbation theory from the effective quark-gluon coupling constant α_s. On the other hand, the dispersion relations express the same polarization operator in terms of an integral of the $e^+e^-\to$hadrons annihilation cross section. This gives rise to the sum rules

$$\frac{1}{\pi}\int_{4m_q^2}^{\infty}\frac{ds\,\mathrm{Im}\,\Pi_{QCD}}{(s+Q^2)^{n+1}}=\frac{1}{\pi}\int_{\mathrm{thresh}}^{\infty}\frac{ds\,\mathrm{Im}\,\Pi_{\mathrm{Phys}}}{(s+Q^2)^{n+1}}, \tag{4}$$

where the left-hand side is obtained from perturbation theory and the right-hand side is expressed in terms of the observed quantities. The index n in relation (4) numbers the order of the derivative of the polarization operator.

The sum rules for the light quarks (u, d, s)[2] are valid if Q^2 is large enough. For the heavy (charmed) quarks we can choose $Q^2=0$. A detailed analysis of the sum rules that occur in the latter case was carried out in[3,4]. It turned out that the first four sum rules $(n=1,2,3)$ agree with experiment within several percent. For $n=5$ the discrepancy is approximately 20% and increases rapidly with the number n.

The existence of such n for which the sum rules at fixed Q^2 are valid is obvious beforehand. Indeed, the physical spectrum contains resonances not accounted for by perturbation theory; their appearance is due to the interaction at large distances and to "dragging" of the quarks.

From the theoretical point of view, the reason why the sum rules do not hold at large n is that operators of nonzero dimensionality contribute to the operator expansion of the two currents

$$i\int T\{j_\mu(x)j_\nu(0)\}e^{iqx}dx=\sum_n C_n O_n, \tag{5}$$

where C_n are numerical coefficients that can be calculated by a series expansion in $\alpha_s(Q)$. The sum rules (4) correspond to the contribution of a single operator having zero dimensionality. The contribution of operators of higher dimensionality (d) is suppressed by the factor $(\mu/Q)^d$ (or $(\mu/2m_q)^d$ for heavy quarks), where μ is of the order of the reciprocal hadron radius. Nonetheless, at fixed Q^2 the power-law corrections increase rapidly with the number of the derivative and become substantial.

The vacuum matrix elements of the operators in (5), with the exception of the unit operator, are equal to zero in perturbation theory and their calculation calls for the use of non-confinement theory (connected with instantons,[5] gauge ambiguity,[6] or some other theory). We propose that the vacuum expectation value (3) differs from zero and consider the phenomenological consequences of this assumption.

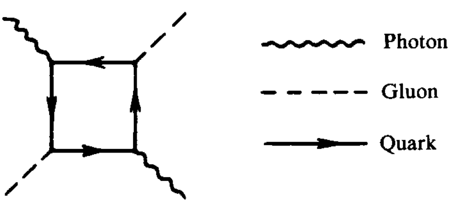
If the vacuum expectation value of (3) differs from zero, then the following increment appears in the polarization operator calculated by perturbation theory:

$$\Delta\Pi = \begin{cases} \dfrac{\alpha_s}{12\pi}\dfrac{\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle}{4Q^4}\left[\dfrac{3(a+1)(a-1)^2}{a^2}\dfrac{1}{2\sqrt{a}}\ln\dfrac{\sqrt{a}+1}{\sqrt{a}-1} - \dfrac{3a^2-2a+3}{a^2}\right] & (\text{heavy quarks}) \\ \dfrac{2m_q\langle 0|\bar{q}q|0\rangle}{Q^4} + \dfrac{\alpha_s}{12\pi}\dfrac{\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle}{Q^4} & (\text{light quarks}) \end{cases} \qquad (6)$$

where $a = 1 + 4m_q^2/Q^2$; $(Q^2 \equiv -q^2)$; m_q is the mass of the deep-virtual quark, while the polarization operator is so normalized that in lowest order of perturbation theory we have

$$\mathrm{Im}\Pi^{(0)} = \frac{1}{4\pi}\,\frac{v(3-v^2)}{2}\,\theta(s - 4m_q^2), \qquad v = (1 - 4m_q^2/s)^{1/2}.$$

The coefficient of the opeator $(G^a_{\mu\nu})^2$ in (6) is determined by diagrams of the following type:



The increment (6) limits the region of applicability of the asymptotic-freedom formulas at large n. Two cases are of interest in applications: light quarks (the limit $m_q \to 0$) and heavy quarks (the mass m_q is large and $Q^2 = 0$). For the ratio of the derivatives of the polarization operator $\Delta\Pi$ and of the operator calculated in zeroth order of perturbation theory we obtain

$$\frac{\left(-\dfrac{d}{dQ^2}\right)^n \Delta\Pi}{\left(-\dfrac{d}{dQ^2}\right)^n \Pi^{(0)}} = \begin{cases} +\, n(n+1)\dfrac{\pi^2}{3}\dfrac{\alpha_s}{\pi}\dfrac{\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle}{Q^4} \quad (\text{light quarks}) & (7) \\ -\,\dfrac{n(n+1)(n+2)(n+3)}{2n+5}\dfrac{4\pi^2}{9}\dfrac{\alpha_s}{\pi}\dfrac{\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle}{(4m_q^2)^2} \quad (\text{heavy quarks}) & (8) \end{cases}$$

It is seen that this ratio increases with increasing n. It is natural to expect the foregoing discrepancy in the sum rules for charmonium at $n = 5$ to be connected with the contribution (6). This assumption fixes the matrix element (3), for which we can then obtain the estimate:

$$\frac{\alpha_s}{\pi}\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle \approx 0.013\ (\mathrm{GeV})^4. \qquad (9)$$

It is important that one and the same matrix element (3) determines for both heavy and light quarks those values of n up to which the asymptotic freedom can be used. From (7) and (8) it is seen that the correction increases more slowly for light quarks, i.e., it is possible to calculate a larger number of derivatives at the same distance from the singularity ($Q^2=4m^2_q$).

Whereas for charmonium the correction is significant at $n=5$, for light quarks it becomes important at $n\sim Q^2/m^2_\rho$. These sum rules are saturated in practice by the contribution of the ρ and ϕ mesons. We then obtain from (4) the relation (1).

The ratio g^2_ϕ/g^2_ρ in (2) depends substantially on the mechanism whereby the SU(3) symmetry is broken. In quantum chromodynamics, the breaking of SU(3) is connected with the difference between the bare masses of the quarks. Consider the difference betwen the polarization operators induced by the currents $j^{(\phi)}=\bar{s}\gamma_\mu s$ and $j^{(\rho)}=(1/\sqrt{2})(\bar{u}\gamma_\mu u-\bar{d}\gamma_\mu d)$. In first order in the SU(3) breaking, this difference is determined by the operator $m_q\bar{q}q$ (q is the field of the quarks u, d, or s) whose vacuum mean value is connected with the spontaneous breaking of chiral symmetry. This can be phenomenologically expressed in terms of the mass and decay constant of the pseudoscalar meson, by using the PCAC hypothesis:

$$\begin{aligned}(m_s + m_u) \langle 0 | \bar{u}u + \bar{s}s | 0 \rangle &\approx -f_K^2 m_K^2 \\ (m_d + m_u) \langle 0 | \bar{u}u + \bar{d}d | 0 \rangle &\approx -f_\pi^2 m_\pi^2 .\end{aligned} \tag{10}$$

By considering a derivative with number $n\sim Q^2/m^2_\rho$ and taking (10) into account, we obtain (2).

We note that the sum rules that take into account the spontaneous breaking of chiral symmetry at small n were discussed in[7]. What is new is the statement that it is permissible to consider a derivative of higher order and to predict the width of the leptonic decay of ϕ.

Another possible application is the calculation of the leptonic width of a vector particle made up of heavier quarks. It is possible that such a meson is Υ with approximate mass 10 GeV.[8] According to (8), in this case asymptotic freedom is valid up to $n\approx 30$. It is interesting that at such values of n the one-resonance contribution again predomainates in the integral of the physical cross section for the production of new particles, and we obtain:

$$\Gamma(\Upsilon \to e^+e^-) \approx 0.5 \text{ keV}, \tag{11}$$

where we have assumed that the quark has a charge $-1/3$. (Sum rules for small n were considered in[9]).

Thus, from the numerical point of view, the statement reduces to meaning that in all the cases considered the sum rules (4) should be valid up to values of n such that saturation by one resonance takes place. The number of derivatives depends in a nontrivial manner on the mass of the quark and is determined from quantum chromodynamics, if the matrix element $\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$ is fixed in a definite manner (e.g., from the sum rules for charmonium).

If the foregoing point of view is correct, then this could be of importance for confinement theory. In particular, operator expansion turns out to be valid not only in the higher-order terms, but, at the very least, in the first power-law correction. Further applications will be considered in succeeding papers.

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