Triton binding energy

D. A. Kirzhnits and N. Zh. Takibaev

P. N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted 28 November 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **27**, No. 1, 73–76 (5 January 1978)

A simple formula is derived for the triton binding energy and yields an estimate $\approx$8 MeV (as against the experimental 8.5 MeV).

PACS numbers: 21.10.Dr, 24.10.Dp, 25.10.+s, 27.10.+h

In our recent papers[1] we have proposed a method of solving the problem of three and more bodies, based on a description of the evolution of the system with changing coupling constant g (gV is the interparticle-interaction potential). In these papers we obtained a solution of the simplest three-body problem—elastic scattering of a neutron (n) by a deuteron (d) in the quartet (spin 3/2) state, and also for the higher orbital momenta of the doublet (spin 1/2) state. We present below the first results pertaining to the doublet s state—calculation of the triton binding energy $|E_t|$.

1. The method is based on the equations for the matrix elements of the potential

$$V'_{\mu\nu} = \sum_{\sigma} V_{\mu\sigma} V_{\sigma\nu} \left[\left(E_\mu - E_\sigma - i\epsilon\right)^{-1} + \left(E_\nu - E_\sigma + i\epsilon\right)^{-1} \right] . \tag{1}$$

The matrix elements diagonal in the energy determine in the discrete spectrum the energy of the bound states

$$E'_\mu = V_{\mu\mu} , \tag{2}$$

while in the continuous spectrum they determine the scattering phase shift δ. In particular, for elastic nd scattering we have

$$\delta'(k) = -\frac{mk}{3\pi} V_{nd,\, nd}(k, k), \tag{3}$$

where m is the nucleon mass, k is the momentum in the c.m.s., the oribital and spin angular momentum indices have been omitted, and the prime denotes differentiation with respect to g.

The solution of the two-nucleon problem using the Yamaguchi split potential

$$-4\pi g\gamma^3/[m(k_1^2+\gamma^2)(k_2^2+\gamma^2)]$$

yields for the deuteron binding energy (4)

$$|E_d| \equiv \kappa^2/m = (g^{1/2}-1)^2\gamma^2/m = 2.23 \text{ MeV}$$

where $\gamma^{-1}=0.69$ F is the effective radius of the forces, $\kappa^{-1}=4.34$ F is the radius of the deuteron, and the physical value of the coupling constant is

$$g = (1+\kappa/\gamma)^2. \tag{5}$$

The derivation of these formulas is contained in[1].

2. The smallness of the quantity $\kappa^2/\gamma^2=0.026$ is a reflection of the "shallowness" of the deuteron level. The same holds also for the triton, where, as will be shown subsequently, the role of $\kappa^2=(-mE_d)$ is played by the quantity

$$X^2 = -\frac{4m}{3}(E_t - E_a), \tag{6}$$

which amounts to one-tenth of γ^2. This enables us to use the pole approximation to solve the system (1), with separation of the strongest singularity in the quantity $(k\pm iX)^{-1}$.

In the matrix element $V_{nd,nd}$, this singularity is obtained directly from the condition that the nd-scattering amplitude has a simple pole at $k=iX$, corresponding to the triton intermediate state. When (3) is taken into account, this yields

$$V_{nd,nd}(k,k) = -\frac{3\pi X^{\prime}}{m}(k^2+X^2)^{-1}+\ldots \tag{7}$$

In general, as seen from (1), the matrix element $V_{\mu\nu}$, whose left (right) index is nd, contains the pole factor $(k+iX)^{-1}$ [or respectively $(k-iX)^{-1}$]. In particular, for the transition into the triton state we have

$$V_{nd,t}(k) = a(k+iX)^{-1}+\ldots, \tag{8}$$

where the constant a will be determined below.

3. We shall henceforth emit the contribution of the three-nucleon intermediate

states in (1), since additional integration with respect to the momenta softens the pole singularity. The triton binding energy is obtained from Eqs. (1), (2), and (8)

$$E_t'' = V_{tt}^{\bullet} = -\frac{8m}{3}\int\frac{d^3k}{k^2+X^2}|V_{t,nd}(k)|^2 = -\frac{m}{3\pi X}|a|^2. \tag{9}$$

On the other hand, Eqs. (1) yield

$$V_{nd,nd}^{\bullet}(k,k) = \frac{8m}{3}\Big[|V_{nd,t}(k)|^2/(k^2+X^2) + \chi d^3p\,|V_{nd,nd}(k,p)|^2/(k^2-p^2)\Big].$$

From this we obtain with the aid of (7) and (8) $|\alpha|^2=(9\pi/2m^2)X(X')^2$, and substitution of this relation in (9), with (4) and (6) taken into account, yields the sought equation for $X(g)$

$$XX'' = \frac{2m}{3}E_d'' = -\gamma^2/(3g^{3/2}). \tag{10}$$

We solve this equation by using the smallness of the ratio κ/γ, i.e., according to (5), the proximity of the coupling constant g to unity. In addition, it was assumed from the very outset that $X^2 \ll \gamma^2$. Such a solution of Eq. (10) does in fact exist

$$X = \sqrt{\frac{2}{3}}\gamma(g-1)[\ln(C/(g-1))]^{1/2}, \tag{11}$$

where C is an arbitrary constant, which can be replaced by unity with logarithmic accuracy. With the same accuracy, using (4)–(6), we obtain the final expression for the triton binding energy

$$E_t/E_d = \ln(\gamma^2/\kappa^2) - 3.68, \tag{12}$$

from which follows in fact that numerical estimate given in the abstract.

4. Expression (11), when understood literally, yields the same triton-production threshold $g=1$ as for the neutron [see (4)]. It is known, however, that a triton is produced at lower values of g, albeit close to unity. The point is that Eq. (1) no longer holds in the immediate vicinity of the point $g=1$, for there is an infinite set of levels that condense at this point (the Efimov effect).[2]

In fact, the solution (11) constitutes more readily the value of X averaged over all the levels. At the physical value of g, when there is only one level, this mean value coincides with the true value of X, and as $g\to 1$ it should actually vanish because of the overwhelming weight of the levels that condense towards zero. With this stipulation, the solution (11), can be used at the point $g=1$ itself.

[1]D.A. Kirzhnits and N. Zh. Takibaev, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 359 (1976) [JETP Lett. **23**, 323 (1976)]; Yad. Fiz. **25**, 700 (1977) [Sov. J. Nucl. Phys. **25**, 370 (1977)].
[2]V.N. Efimov, Yad. Fiz. **12**, 1080 (1970) [Sov. J. Nucl. Phys. **12**, 589 (1971)].