FEASIBILITY OF EXPERIMENTAL VERIFICATION OF THE CONFORMAL INVARIANCE HYPOTHESIS

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It is shown that the number of particles and the energy density are linear combinations of quantities that have definite scale dimensionalities near the liquid-vapor critical point. It is proposed to verify an important consequence of the conformal-invariance hypothesis, viz., the orthogonality of quantities with different scale dimensionalities, i.e., the vanishing of the pair correlator of such quantities.

The conformal hypothesis of invariance (CI) in the theory of phase transitions was advanced by A.M. Polyakov [1]. It had appeared so far that the experimentally measured thermodynamic quantities could not yield any information on the conformal invariance of the fluctuation field $\phi(x)$. Scattering experiments are likewise unsuitable as checks on the CI, since they yield only information on the pair correlation function $\langle\phi(x)\phi(x')\rangle$. Yet the form of this correlation function is determined uniquely from considerations of homogeneity, isotropy, and scale invariance. The measurement of triple correlators whose form is determined by the CI is hardly possible.

An important consequence of CI is the "orthogonality" relation for the quantities O_1 and O_2 having different scale dimensionalities $\Delta_1 \neq \Delta_2$:

$$\langle\langle O_1 O_2 \rangle\rangle \equiv \langle O_1 O_2 \rangle - \langle O_1 \rangle \langle O_2 \rangle = 0 \tag{1}$$

To assess the possibility of verifying (1), we perform a dimensional analysis of the physical quantities near the liquid-vapor critical point.

We note first the important difference between a real liquid or gas and the model of a lattice gas (see, e.g., [2]). In a lattice gas the fluctuations of the energy E and of the number of particles N are statistically independent on the critical isochore

$$\langle \Delta N \, \Delta E \rangle |_{V=V_c} = 0 \tag{2}$$

This equality is connected with the special symmetry of the lattice model. It is known that this model is isomorphic to the Ising model. Equation (2) is the analog of the obvious relation

$$\langle \Delta M \;\; \Delta E \rangle |_{h=0} = 0, \tag{3}$$

where M is the total moment and h is the magnetic field in the Ising model. In a real liquid-vapor system, ΔN and ΔE are not independent:

$$\langle \Delta N \;\; \Delta E \rangle = \left(\frac{\partial E}{\partial N}\right)_{V,T} \langle (\Delta N)^2 \rangle = \left[\mu + Ts - Tv\left(\frac{\partial p}{\partial T}\right)_V\right]\left(\frac{\partial N}{\partial \mu}\right)_{T,V}. \tag{4}$$

Here μ is the chemical potential, s the entropy, and v the volume, all per particle. The quantity in the square bracket is by no means equal to zero on the critical isochore.

Equation (4) means that ΔE is just as strongly fluctuating a quantity as ΔN. Let O_1 and O_2 be two singular additive operators with positive definite dimensionalities Δ_1 and Δ_2. The latter means that

$$\langle O_i^n \rangle \sim V r_c^{-n\Delta_i - 3} \qquad (\Delta_1 < \Delta_2 < 0) \tag{5}$$

Here r_c is the correlation radius. We assume also that the dimensionalities of the other algebra operators of the fluctuating quantities are $\Delta_k > \Delta_2$ ($k = 3, 4, \ldots$). We represent ΔE and ΔN in the form of a linear combination of algebra operators, neglecting all the quantities with the exception of O_1 and O_2:

$$\begin{aligned} \Delta N &= a O_1 + b O_2 \\ \Delta E &= a' O_1 + b' O_2 \end{aligned} \qquad (6)$$

In the principal order, such thermodynamic quantities as the compressibility, the thermal coefficient of volume expansion, and the heat capacity C_{V_μ} are determined by the fluctuations $\langle O_1^2 \rangle$ of the quantity O_1 and are proportional to one another. The experimentally measured quantity is the heat capacity C_{VN} and not C_{V_μ}. It is surprising that the singularity of C_{VN} is connected with fluctuations of O_2 and not of O_1! In fact, let us express C_{VN} with the aid of the potential $\Omega = -pV$:

$$C_{VN} = T \frac{\Omega_{\mu T}^2 - \Omega_{\mu\mu} \Omega_{TT}}{\Omega_{\mu\mu}} \qquad (7)$$

(the subscripts μ and T of Ω denote differentiation). The second derivatives of Ω are equal to the fluctuation mean values:

$$\Omega_{\mu\mu} = -\langle (\Delta N)^2 \rangle; \quad \Omega_{TT} = -\langle (\Delta \tilde{E})^2 \rangle; \quad \Omega_{\mu T} = -\langle \Delta \tilde{E} \cdot \Delta N \rangle \qquad (8)$$
$$\tilde{E} = E - \mu N .$$

Substituting (8) and (6) in (7), we find that the coefficients of $\langle O_1^2 \rangle$ and $\langle O_1 O_2 \rangle$ in C_{VN} vanish identically[1]. and in the first nonvanishing approximation we have

$$C_{VN} = \frac{(ab' - a'b)^2}{a^2} \langle O_2^2 \rangle \qquad (9)$$

The singularity of C_{VN} turned out to be weak in comparison with C_{pN} and with the compressibility, as in the lattice-gas model and in the experiments [4].

We use the well-known relation

$$\left(\frac{\partial n}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_n + \left(\frac{\partial n}{\partial T}\right)_p = 0 \qquad (10)$$

It follows from it, in particular, that $(\partial p/\partial T)_n$ also assumes a finite value at the critical point:

$$\left(\frac{\partial p}{\partial T}\right)_n = \left(s_c - \frac{a'}{a}\right) n_c \qquad (11)$$

We replace in the left-hand side of (10) the quantity $(\partial p/\partial T)_n$ by its value (11) at the critical point. The obtained quantity no longer vanishes identically,

[1] A similar cancellation takes place also in the case of the point of He (see [3]).

but the principal singularities proportional to $\sim\langle O_1^2\rangle$ are subtracted. After simple operations we obtain

$$\left(\frac{\partial n}{\partial p}\right)_T\left(\frac{\partial p}{\partial T}\right)_{Cr}+\left(\frac{\partial n}{\partial T}\right)_p=\left[(s_c-s)a^2+\frac{n-n_c}{n}aa'\right]\langle O_1^2\rangle+$$

$$+\frac{ab'-a'b}{a}\langle O_1O_2\rangle+\frac{b}{a}(ab'-a'b)\langle O_2^2\rangle .$$

Dimensionally, $\langle O_1^2\rangle\sim\tau^{-\gamma}$, $\langle O_2^2\rangle\sim\tau^{-\alpha}$, and $s_c - s\sim\tau^{1-\alpha}$. Finally, $\langle O_1O_2\rangle\sim[\langle O_1^2\rangle\langle O_2^2\rangle]^{1/2}\sim\tau^{-(\alpha+\gamma)/2}$ if this mean value differs from zero. Putting $\gamma\simeq 1.25$ and $\alpha\simeq 0.12$, we find that the order of magnitude of (12) ($\sim\tau^{-0.7}$) is determined by the second term if $\langle O_1O_2\rangle\neq 0$. Otherwise the first term becomes principal ($\sim\tau^{-0.4}$). The measurements should be made along an isochore very close to $n = n_c$, or else the term with $\langle O_1^2\rangle$ will become more significant. The relative measurement error of all the quantities should not exceed $\tau^{-(\gamma-\alpha)/2}\sim\tau^{-0.5}$.

[1] A.M. Polyakov, ZhETF Pis. Red. 12, 538 (1970) [JETP Lett. 12, 381 (1970)].

[2] M. Fisher, The Nature of Critical Phenomena (Russ. transl.), Mir.

[3] E.G. Batyev, A.Z. Patashinskii, and V.L. Pokrovskii, Zh. Eksp. Teor. Fiz. 47, 598 (1964) [Sov.Phys.-JETP 20, 398 (1965)].

[4] M.N. Bagatskii, A.V. Voronel', and V.G. Gusak, ibid. 43, 728 (1962) [16, 517 (1963)]. V.L. Pokrovskii, Usp. Fiz. Nauk 94, 127 (1968) [Sov. Phys.-Usp. 11, 66 (1968)].

LEPTON MODEL

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The maximal group $SU(2)\otimes U(1)$, which unifies weak and electromagnetic interactions of known leptons in a renormalizable gauge theory with spontaneously broken symmetry [1], predicts the existence of weak interactions of neutron currents, with obligatory participation of a neutrino current in the form $\bar{\nu}\gamma_\alpha(1+\gamma_5)\nu$. The experimental data seem to contradict this prediction [2]. The need for a hypothesis calling for the existence of heavy leptons in order to eliminate from the weak interactions the symmetric neutral neutrino currents within the framework of the scheme with violated isotopic properties of the leptons was first indicated in [3]. Quite recently, a similar idea was used in an interesting model by Georgi and Glashow [4] for complete elimination of weak interaction of neutral currents in a renormalizable theory of type [1], but with the $SU(2)\otimes U(1)$ group replaced by $O(3)$.

We call attention in this article to a logically simple possibility of unifying Weinberg's initial $SU(2)\otimes U(1)$ symmetrical lepton model [1] with the classification given in [3] for the family of leptons, such that the interactions of the $(\bar{\nu}O_\alpha\nu)$ current are naturally eliminated together with the difficulties with the triangular anomalies of the axial current. We shall classify the massless leptons with respect to the invariants of the fundamental (doublet) representations of the SU(2) group: lepton charge ℓ, hypercharge Y, and helicity (the eigenvalues of the operator γ_5). In Table I are marked 4 of the 8