but the principal singularities proportional to $\sim\langle O_1^2\rangle$ are subtracted. After simple operations we obtain

$$\left(\frac{\partial n}{\partial p}\right)_T\left(\frac{\partial p}{\partial T}\right)_{Cr} + \left(\frac{\partial n}{\partial T}\right)_p = \left[(s_c - s)a^2 + \frac{n - n_c}{n} a a'\right]\langle O_1^2\rangle +$$

$$+ \frac{ab' - a'b}{a}\langle O_1 O_2\rangle + \frac{b}{a}(ab' - a'b)\langle O_2^2\rangle .$$

Dimensionally, $\langle O_1^2\rangle \sim \tau^{-\gamma}$, $\langle O_2^2\rangle \sim \tau^{-\alpha}$, and $s_c - s \sim \tau^{1-\alpha}$. Finally, $\langle O_1 O_2\rangle \sim [\langle O_1^2\rangle\langle O_2^2\rangle]^{1/2} \sim \tau^{-(\alpha+\gamma)/2}$ if this mean value differs from zero. Putting $\gamma \approx 1.25$ and $\alpha \approx 0.12$, we find that the order of magnitude of (12) ($\sim\tau^{-0.7}$) is determined by the second term if $\langle O_1 O_2\rangle \neq 0$. Otherwise the first term becomes principal ($\sim\tau^{-0.4}$). The measurements should be made along an isochore very close to $n = n_c$, or else the term with $\langle O_1^2\rangle$ will become more significant. The relative measurement error of all the quantities should not exceed $\tau^{-(\gamma-\alpha)/2} \sim \tau^{-0.5}$.

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LEPTON MODEL

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The maximal group SU(2)⊗U(1), which unifies weak and electromagnetic interactions of known leptons in a renormalizable gauge theory with spontaneously broken symmetry [1], predicts the existence of weak interactions of neutron currents, with obligatory participation of a neutrino current in the form $\bar{\nu}\gamma_\alpha(1 + \gamma_5)\nu$. The experimental data seem to contradict this prediction [2]. The need for a hypothesis calling for the existence of heavy leptons in order to eliminate from the weak interactions the symmetric neutral neutrino currents within the framework of the scheme with violated isotopic properties of the leptons was first indicated in [3]. Quite recently, a similar idea was used in an interesting model by Georgi and Glashow [4] for complete elimination of weak interaction of neutral currents in a renormalizable theory of type [1], but with the SU(2)⊗U(1) group replaced by O(3).

We call attention in this article to a logically simple possibility of unifying Weinberg's initial SU(2)⊗U(1) symmetrical lepton model [1] with the classification given in [3] for the family of leptons, such that the interactions of the $(\bar{\nu}O_\alpha\nu)$ current are naturally eliminated together with the difficulties with the triangular anomalies of the axial current. We shall classify the massless leptons with respect to the invariants of the fundamental (doublet) representations of the SU(2) group: lepton charge ℓ, hypercharge Y, and helicity (the eigenvalues of the operator γ_5). In Table I are marked 4 of the 8

Table 1

Symbol	ℓ	Y	γ_5
L_{1L}	+1	−1	+1
L_{2L}	−1	−1	+1
L_{3L}	+1	+1	+1
L_{4L}	−1	+1	+1

Table 2

Symbol	ℓ	Q	γ_5
e_R^-	+1	−1	−1
μ_R^-	−1	−1	−1
$e_R'^+$	+1	+1	−1
$\mu_R'^+$	−1	+1	−1

Table 3

Symbol	ℓ	γ_5
e_R^0	+1	−1
μ_R^0	−1	−1

resultant doublets, and the remaining correspond to antiparticles with opposite signs of all the quantum numbers. We introduce the notation

$$L_{1L} = \begin{pmatrix} \ell^0_{1L} \\ e^-_L \end{pmatrix}, \quad L_{2L} = \begin{pmatrix} \ell^0_{2L} \\ \mu^-_L \end{pmatrix}, \quad L_{3L} = \begin{pmatrix} e'^+_L \\ \ell^0_{3L} \end{pmatrix}, \quad L_{4L} = \begin{pmatrix} \mu'^+_L \\ \ell^0_{4L} \end{pmatrix}, \tag{1}$$

where the subscripts L and R for any operator a are determined from the rule $a_{L,R} = \frac{1}{2}(1 \pm \gamma_5)a$. All these doublets can be combined into one eight-component lepton operator

$$\Psi_L = \begin{pmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{pmatrix}_L, \qquad \tilde{\Psi}_R = \begin{pmatrix} \tilde{L}_1 \\ \tilde{L}_2 \\ \tilde{L}_3 \\ \tilde{L}_4 \end{pmatrix}_R , \tag{2}$$

where the tilde denotes the antiparticles. Ψ_L realizes the fundamental representation of the SU(2) group, which is the direct sum of four fundamental representations in the spaces of each doublet. This circumstance is the formulation of the universality property of weak and electromagnetic lepton interactions. We write the generators of the SU(2) ⊗ U(1) group in the form

$$T_i = \frac{1}{4}(1 + \gamma_5)\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \otimes \tau_i , \quad Y = \frac{1}{4}(1 + \gamma_5)\begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \otimes I + Q_R , \tag{3}$$

where i = 1, 2, 3, τ_i are Pauli matrices, I is a unit 2 × 2 matrix, Q_R is the electric-charge operator of the "right-hand" particles, and

$$Q = Q_L + Q_R = T_3 + Y. \tag{4}$$

It is obvious that the operators (3) satisfy the permutation relations of SU(2) ⊗ U(1) algebra. The system of four lepton currents occurs after the generators (3) are bracketed

$$j_{i\alpha} = \bar{\Psi}_L \gamma_\alpha T_i \Psi_L , \qquad j_{Y\alpha} = \bar{\Psi}_L \gamma_\alpha Y \Psi_L + (J_\alpha^{\text{ЭМ}})_R , \tag{5}$$

where $(J_\alpha^{em})_R$ is the electromagnetic current of the right-hand leptons listed in Table 2. The SU(2) ⊗ U(1)-invariant interaction of the lepton currents with the vector gauge fields is of the form

$$ig \sum_{i=1}^{3} j_{i\alpha} A_{i\alpha} + ig' j_{Y\alpha} B_\alpha . \tag{6}$$

The reason for the appearance of particle masses here, as in [1], is only the spontaneous violation of the symmetry, which is connected with the nonzero mean vacuum values of the scalar fields. The lepton masses appear as a result of the trilinear SU(2)⊗U(1)-invariant terms of the interaction Lagrangian of the lepton doublets (1) with the doublets of the scalar fields and with the singlet lepton states, which are not contained in Table 1. We postulate that these singlet states correspond to a classification with respect to all possible eigenvalues of the operators Q, ℓ, and γ_5 (Table 2) for charged leptons, and with respect to the values of ℓ and γ_5 for neutral leptons (Table 3); in the minimal model, each combination of quantum numbers corresponds to only one singlet state. This means that in such a model all the charge leptons can acquire nonzero masses as through the mechanism of spontaneous symmetry breaking, but this mechanism can generate masses only for two neutral leptons out of the four contained in the system (1). Two two-component neutral leptons should not enter in the mass complexes, and it follows from Table 1 that they can always be unified into one four-component spinor. Consequently, the present model leads in natural fashion to a scheme of universal weak interaction with one four-component neutrino and with maximum parity nonconservation.

We note now that, as follows from a comparison of Tables 3 and 1, the choice of two out of four neutral L-states of the system (1), which together with E_R^0 and μ_R^0 could form two massive neutral leptons e^0 and μ^0, is not unique. We make this choice in such a symmetrical manner as to ensure universality of weak neutrino interactions. It is seen from the structure of the generators (3) that to this end it suffices to determine

$$\ell_{1L}^{0} = \frac{1}{\sqrt{2}}(\nu + e^{0})_L, \quad \ell_{2L}^{0} = \frac{1}{\sqrt{2}}(\tilde{\nu} + \mu^{0})_L, \quad \ell_{3L}^{0} = \frac{1}{\sqrt{2}}(\nu - e^{0})_L, \quad \ell_{4L}^{0} = \frac{1}{\sqrt{2}}(\tilde{\nu} - \mu^{0})_L . \tag{7}$$

It is easy to verify that in this case the symmetrical neutrino current $(\bar{\nu}0_\alpha\nu)$ is automatically excluded from the theory. Indeed, in full accord with Weinberg's initial scheme [1], we obtain from (6) the following expression for the Lagrangians of the weak and electromagnetic lepton interactions.

$$ieJ_\alpha^{em}A_\alpha + i\frac{g}{2\sqrt{2}}J_\alpha^W W_\alpha + h.c. + \frac{i}{4}(g^2 + g'^2)^{1/2}J_\alpha^z Z_\alpha , \tag{8}$$

where $e \equiv gg'(g^2 + g'^2)^{-1/2}$ is the electric charge, J_α^{em} is the electromagnetic lepton current, A_α is a photon, W_α is the charged intermediate weak-interaction boson, Z_α is a heavy neutral boson,

$$J_\alpha^W = 2^{-1/2}(\bar{\nu}_e O_\alpha e^- + \bar{\nu}_\mu O_\alpha \mu^- + \ldots), \tag{9}$$

where $0_\alpha = \gamma_\alpha(1 + \gamma_5)$, $\nu_e \equiv \nu_L$, $\nu_\mu \equiv \tilde{\nu}_L$, the dots stand for the obvious terms with the heavy leptons, and

$$J_\alpha^Z = \bar{\nu}_e O_\alpha e^0 + \bar{\nu}_\mu O_\alpha \mu^0 - h.c. - \left(\frac{3g'^2 - g^2}{g^2 + g'^2}\right) J_\alpha^{em} + J_{5\alpha}^{em} \tag{10}$$

where the axial current $J_{5\alpha}^{em}$ is obtained from J_α^{em} by substituting $\gamma_\alpha \to \gamma_\alpha\gamma_5$. The currents (9) and (10) describe the universal interaction of massive leptons with neutrinos and do not contain the term $(\bar{\nu}0_\alpha\nu)$. It is easy to verify that the characteristic structure of the currents J_α^{em} and $J_{5\alpha}^{em}$ in this model ensures cancellation of the triangular anomalies of the axial currents. For spontaneous symmetry breaking one can use one doublet of scalar fields, and no limitations are imposed on the masses of the heavy charged and neutral leptons $m_{e'}$, $m_{\mu'}$, m_{e^0}, and m_{μ^0}. However, the masses of the intermediate vector bosons must satisfy the inequalities

$$M_W > 26\ \text{GeV}, \quad M_Z > 52\ \text{Gev}. \tag{11}$$

After writing this article, the author has learned that the concrete model of weak and electromagnetic lepton interaction considered here coincides essentially with model No. 2 of Prentki and Zumino [5]. The purpose of the present communication is therefore mainly a discussion of the possible "reasonable basis" for the family of leptons, which apparently singles out this model from the among the many other possible models with heavy leptons. The lepton classification scheme considered here determines seven four-component lepton spinors, which describe six massive particles and one massless neutrino. This classification is peculiar to leptons and does not admit of a straight analogy between hadrons and leptons.

The attractive features of the theory of a single renormalizable electromagnetic-weak interaction, in view of the negative results of the searches for $(\bar{\nu}O_\alpha\nu)$-current interactions, gives apparently certain preference to the scheme of interaction of heavy leptons with the known neutrinos ν_e and ν_μ as against the schemes where the heavy leptons interact only with their proper neutral partners. The predictions of this scheme, which are based on expressions (9) and (10) for weak currents, where one neutrino interacts universally with all six massive leptons, were formulated in [3, 6, 7] and have been recently widely discussed in the literature [8]. Some theoretical arguments favoring such a scheme were presented above.

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