Dye	λ_{gen}, nm	Concentr. mol/l	Eff.% (max)	E_{out}, J at max eff.	E_{out}, J at E_{pump} = 1,5 kJ;	P_{out}, MW
Cresyl violet	650	10^{-4}	0,11	1,4	1,6	2
Rhodamine-6G	590	10^{-4}	1,1	12	15	7,5
Rhodamine unsubstituted	560	10^{-4}	0,03	0,3	0,4	0,6
7-oxy-4-methyl-3-ethylcoumarin	460	$5\cdot 10^{-4}$	0,55	6	7.6	4
4-methylumbelliferone (4MU)	455	$5\cdot 10^{-4}$	1,25	12,5	14,5	7,25
7-amino-4-methylcoumarin	435	$5\cdot 10^{-4}$	0,13	2	2	2,5

[1] S. A. Mikhnov, M. I. Zybin, and V. S. Strizhnev, Zh. Prikl. Spektr. 15, No. 5, 947(1971).
[2] F. N. Baltakov, V. A. Barikhin, V. G. Kornilov, S. A. Mikhnov, A. N. Rubinov, and L. V. Sukhanov, Zh. Tekh. Fiz. 42, 1459 (1972) [Sov. Phys.-Tech. Phys. 17, 1161 (1973)].
[3] M. Bass, T. F. Deutsch, and M. J. Weber, Tech. Report R-69, Raytheon Co., Waltham, Mass. Published also in : Lasers, Vol. 3, NY, 1971.
[4] Yu. N. Yanait, G. A. Abakumov, G. I. Kromskii, A. P. Simonov, V. V. Fadeev, and R. V. Khokhlov, ZhETF Pis. Red. 13, 616 (1971) [JETP Lett. 13, 438 (1971)]
[5] M. I. Dzyubenko, A. Ya. Matveev, and I. G. Naumenko, Prib. Tekh. Eksp. No. 1, 171 (1972).

CONTRIBUTION TO THE NONLINEAR THEORY OF THE "MODIFIED" DECAY INSTABILITY

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An expression is derived for heat flow in a plasma in which the electron mean free path exceeds the characteristic scale of the inhomogeneity. This expression is used to derive a formula for the electron temperature in a plasma corona as a function of the laser power.

We investigate the nonlinear stage of the "modified" decay [1] of a Langmuir wave into a Langmuir satellite and a low-frequency (LF) perturbation of the acoustic type, when the instability increment greatly exceeds the frequency of the sound and the perturbation grows aperiodically with time.

The nonlinearity mechanism considered by us consists in the reaction of the pump wave on the growth of the perturbation. In ordinary decay, such a mechanism leads to establishment of a equilibrium state in which the energy of the wave motions is transferred mainly between the high-frequency modes (the energy in the sound wave is smaller in the ratio ω_S/ω_L) [2]. We shall show that in the case of an aperiodic instability no such oscillating equilibrium is established, and the amplitude of the LF perturbations increases to an appreciably larger value, at which considerable modulation of the plasma density takes place. Under these conditions, irreversible dissipation of the pump-wave energy becomes possible and the electrons and ions can be heated.

The system of equations describing the "modified" decay of the Langmuir waves is written in the form[1)]

$$\frac{d}{d\tau}(C_0 e^{i\alpha_0}) = -i\, C_1 S\, e^{i(\alpha_1 - \Delta_1\tau - \phi)}, \qquad (1)$$

$$\frac{d}{d\tau}(C_1 e^{i\alpha_1}) = -iC_o S e^{i(\alpha_o + \Delta_1 \tau + \phi)}, \qquad (2)$$

$$\frac{d^2}{d\tau^2}(S e^{i\phi}) + \Gamma S e^{i\phi} = -C_o C_1 e^{i(\alpha_1 - \alpha_o - \Delta_1 \tau)}. \qquad (3)$$

In these equations we have used the dimensionless variables

$$C_i = \frac{E_i(t)}{E_o(t=0)}, \quad S = \frac{n_s(t)}{2n_o \Lambda}, \quad \tau = \frac{1}{2}\omega_o \Lambda t, \qquad (4)$$

where $\Lambda = (m_e/m_i)^{1/3}(v_E/v_{ph})^{2/3}$ and $\Gamma = (2\omega_s/\omega_0\Lambda)$ are parameters, $v_E = eE_0(t = 0)/m_e\omega_0$ is the velocity of the electrons in the Langmuir wave, $v_{ph} = \omega_0/k_0$, $E_j(t)$ and $\alpha_j(t)$ are the amplitude and phase of the Langmuir waves ($j = 0, 1$); $E_j(t, z) = \frac{1}{2}E_j(t) \exp[ik_j z - \omega_j t + \alpha_j(t)] + c.c.$, $\omega_j = (\omega_L^2 + 3k_j^2 v_T^2)^{1/2}$ is the frequency, the "detuning" is $\Delta_j = (\omega_j^2 - \omega_0^2)/(\Lambda\omega_0^2)$; $n_s(t)$ is the amplitude of the quasineutral density perturbation in the low-frequency mode: $n_s(t, z) = \frac{1}{2}n_s(t) \exp[i(kz + \phi(t))] + c. c.$

We have neglected the excitation of the Langmuir satellite with wave number $k_{-1} = k_0 - k$, assuming that the "detuning Δ_{-1} is large enough. In the approximation with a given pump wave, putting $Se^{i\phi} \sim e^{i\nu\tau}$, we get from (2) and (3) the dispersion equation of the linear theory

$$(\nu - \Delta_1)(\nu^2 - \Gamma) = 1. \qquad (5)$$

This equation describes the continuous transition when Γ decreases from the parameteric excitation of ion sound, investigated in [3], to the "modified" decay, the condition for the onset of which is $\Gamma \ll 1$, i.e., $v_E^2/v_T^2 \gg (m_e/m_i)^{1/2} v_T/v_{ph}$. As $\Gamma \to 0$, the instability develops for all $\Delta_1 > -3/2^{2/3}$.

Equations (1) - (3) are valid in the energy interval

$$H = (\dot{S})^2 + S^2(\dot{\phi})^2 + \Gamma S^2 + \omega_o C_o^2 + \omega_1 C_1^2 + 2SC_o C_1 \cos(\alpha_o - \alpha_1 + \phi + \Delta_1 \tau) \qquad (6)$$

There exist also integral analogous to the Manley-Rowe relations [2]:

$$C_o^2 + C_1^2 = \text{const}, \quad 2S^2\dot{\phi} - C_1^2 = \text{const}. \qquad (7)$$

An analytic solution of the nonlinear problem can be obtained when $\Delta_1 \gg 1$. In this case, according to (5), there exists an aperiodic instability with increment $\text{Im}\,\nu \sim 1/\sqrt{\Delta_1} \ll \Delta_1$, and accordingly the amplitude of the Langmuir perturbation varies with time much more slowly than the phase. This enables us to integrate (1) and (2) and obtain the following relations for the amplitudes and phases of the Langmuir waves:

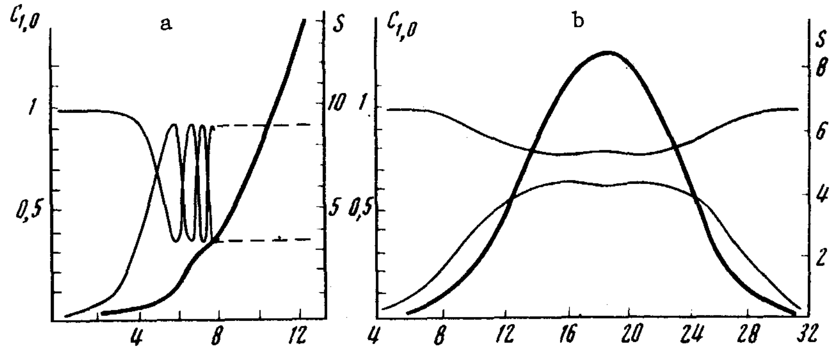
$$\alpha_o - \alpha_1 + \phi + \Delta_1 \tau = \pi, \quad \dot{\alpha}_o \dot{\alpha}_1 = S^2, \quad C_1 = \frac{SC_o}{\dot{\alpha}_1}, \quad C_o^2\left(1 + \frac{S^2}{\dot{\alpha}_1^2}\right) = 1. \qquad (8)$$

For the amplitude of the LF perturbation, using (7) - (8) and integrating with respect to τ, we obtain the following equation:

$$\left(\frac{dS}{d\tau}\right)^2 = \left(\frac{dS_o}{d\tau}\right)^2 + \sqrt{\frac{\Delta_1^2}{4} + S^2} - \sqrt{\frac{\Delta_1^2}{4} + S_o^2} - \Gamma(S^2 - S_o^2), \qquad (9)$$

where we have used the notation $S_0 = S(\tau = 0)$ and $dS_0/d\tau = dS/d\tau$ ($\tau = 0$). As $\Gamma \to 0$, the function $S(\tau)$ determined from this equation increases without limit with time: $S \simeq \tau^2/4$ at $S \gg 1$. Allowance for the term $\sim\Gamma$ in the equation leads to a periodic solution with period

$$\tau_o = \frac{2\pi}{\sqrt{\Gamma}} + 2\sqrt{\Delta_1}\, K\left(1 - \frac{|a|}{\Delta_1}\right), \quad a = \frac{S_o^2}{\Delta_1} - \left(\frac{dS_o}{d\tau}\right)^2$$



($K(\kappa)$ is a complete elliptic integral of the first kind), in which S ranges from a minimal value $S_{min} \simeq \sqrt{\alpha\Delta_1}$ ($S_{min} = 0$ at $\alpha < 0$) to a maximum $S_{max} \simeq 1/\Gamma$, corresponding to the density perturbation $n_{S\ max} = n_0 V_E^2/2v_T^2$ in the low-frequency mode. The maximum value of the energy transferred to the ions also turns out to be quite appreciable:

$$W_{i\ max} = n_o \frac{m_i v_{i\ max}^2}{2} \approx n_o m_e v_E^2 \frac{v_E^2}{v_T^2} . \qquad (10)$$

The solution of the initial equations (1) - (3) for arbitrary "detuning" Δ_1 can be obtained by numerical means. At $\Delta_1 \geq 3$, the solution obtained in this manner coincides with great accuracy with the analytic one; by way of example, Fig. 1b shows the results of numerical integration of Eqs. (1) - (3) at $\Gamma = 0.1$ and $\Delta_1 = 3$. In the case $\Gamma = 0$, S continues to grow without limit with time at all $\Delta_1 \geq 0$ (see Fig. 1a, for which $\Gamma = 0$ and $\Delta_1 = 0$).

Our analysis is valid if $v_E^2 \ll v_T^2$. Increasing the pump-wave amplitudes to values such that $v_E/v_T \gtrsim 1$ makes it necessary to take into account the electronic nonlinearity in the initial equations. The qualitative situation is then as follows: The increase of the amplitude of the low-frequency wave produces at $S \sim 1/\Lambda$ a breaking of the wave front and a crossing of the particle trajectories. The solution becomes irreversible and the energy of the pump wave is dissipated on the electrons and ions.

The singularities obtained here for the nonlinear solution describing the "modified" decay remain in force also in the case of four-wave parametric instability (j = 0.1 - 1) [1,4], which we shall consider separately.

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[1)] The dimensionless equations (1) - (3) obviously describe a "modified" decay of a pump wave of arbitrary character, and in particular an electromagnetic wave.

[1] A. A. Galeev and R. Z. Sagdeev, Nucl. Fusion 13, No. 4 (1973).
[2] N. Bloembergen, Nonlinear Optics, Benjamin, 1965.
[3] V. N. Oraevskii and R. Z. Sagdeev, Zh. Tekh. Fiz. 32, 1291 (1962) [Sov. Phys.-Tech. Phys. 7, 955 (1963)].
[4] N. E . Andreev, A. Yu. Kirii, and V. P. Silin, Zh. Eksp. Teor. Fiz. 57, 1024 (1969) [Sov. Phys.-JETP 30, 559 (1970)].