equal to

$$T_c \simeq \mu . \quad (10)$$

The stability of the considered superconducting state and the role of multiphonon processes in the kinetic phenomena call for a separate study.

The authors are grateful to Yu. A. Bykovskii, L. V. Keldysh, and D. A. Kirzhnits for useful remarks and discussions.

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1) Analogous considerations were advanced also by A. G. Aronov and V. L. Gurevich.

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SPONTANEOUS POSITRON PRODUCTION BY THE COULOMB FIELD OF A HEAVY NUCLEUS

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Submitted 21 May 1973
ZhETF Pis. Red. 18, No. 1, 53 - 58 (5 July 1973)

The spontaneous production of positrons in a Coulomb field with $Z > Z_c$ (where $Z_c \simeq 170$ is the critical charge of the nucleus [1 - 3] is of interest as an example of e^+e^- pair production via an unusual static mechanism [4 - 9]. The most realistic way of searching for this phenomenon is in collisions of heavy nuclei with $Z_1 + Z_2 > Z_c$, for example uranium or californium nuclei. It is not necessary that both colliding nuclei be "bare" (i.e., have no electron shells). When a beam of bare nuclei Z_1 is incident on an ordinary heavy target Z_2, vacant places are produced in the K shell of the combined atom if $Z_1 > Z_2$. The cross section σ for the production of e^+ has in this case practically the same value as for bare nuclei [8, 9].

A detailed discussion of this process, as well as an estimate of the background effects, can be found in [9]. There, however, the cross section σ was calculated using a threshold formula valid only for $E \to E_t$ (i.e., where the cross section itself is exponentially small). In view of the strong dependence of σ on the energy E of the colliding nuclei, extrapolation of this formula to the region $E \gtrsim 1.5E_t$ is not valid[1]. At the same time, calculation of the cross section σ in the entire range of energies and the energy spectrum of the produced positrons is needed for an experiment on the spontaneous production of e^+ in nuclear collisions, which is being planned for the near future. The results of such a calculations are presented below.

Since the velocity of the nuclei is much lower than the velocity of the electron on the K orbit (at $Z\alpha \sim 1$), the process of e^+ production can be regarded in the adiabatic approximation. The cross section σ is then expressed in terms of the imaginary part γ of the energy of the quasistationary level that has gone to the lower continuum at $R < R_c$ ($\varepsilon = \varepsilon_0 + i\gamma/2$, $\varepsilon_0 < -1$). Exact calculation of ε_0 and γ as functions of the internuclear distance R calls for a solution of the Dirac equation for the two-center problem and entails considerable mathematical difficulties. We have used the fact that in the region $Z \sim 90 - 100$ there is satisfied the condition of "low supercriticality"

$$\delta = (Z_1 + Z_2 - Z_c)/Z_c << 1 \quad (1)$$

(we put for simplicity $Z_1 = Z_2 = Z$). Consequently, the solution of the relativistic two-center problem can be obtained in analytic form by using the method of matching the asymptotic forms [10]. The energy ε of the ground level of the quasimolecule $1s\sigma$ is determined by the asymptotic equation

$$\ln \frac{R_c}{R} = \psi\left(-\frac{\epsilon}{\lambda}\right) + \ln \lambda + \frac{1+\epsilon}{1+\epsilon+\lambda} , \quad (2)$$

which is valid if $R_c \ll \hbar/m_e c = 1$. Here $\lambda = (1-\varepsilon^2)^{1/2}$, $\psi(z) = \Gamma'(z)/\Gamma(z)$, and R_c is the critical distance between nuclei, at which $\varepsilon(1s\sigma) = -1$. We note that the charge Z enters here only via R_c, i.e., the level motion possesses a self-similar character of sorts:

$$\varepsilon = \varepsilon(R/R_c). \tag{3}$$

This is the consequence of approximation (1), under which R_c and R are small in comparison with the average radius of the K orbit when $Z\alpha \sim 1$.

At $R < R_c$ we put in (2) $\lambda = -ik$, as a result of which ε acquires an imaginary part, and $\gamma \ll |\varepsilon_0|$ at all times[2]. Taking this into account, we can transform (2) into

$$\ln\frac{R_c}{R} = \mathrm{Re}\ \psi(iy) + \frac{1}{2}[1-\sqrt{1-y^{-2}} - \ln(y^2-1)] \tag{4}$$

$$\gamma = \frac{2\pi k^3}{(e^{2\pi y}-1)[\mathrm{Im}\,\psi'(iy) + k^2 y + k/2y^2]}, \tag{5}$$

where $k = \sqrt{\varepsilon_0 - 1}$ and $y = \sqrt{1+k^2}/k$. Equation (4) determines, given R, the resonance energy $\varepsilon_0 = -y/\sqrt{y^2-1}$ and the momentum k of the emitted electrons, after which formula (5) determines $\gamma = \gamma(R)$. The corresponding curves are shown in Figs. 1 and 2.

We note that the curve for $\varepsilon_0(R)$ has no kink at the critical point $R = R_c$, and the singularity of the function $\varepsilon = \varepsilon(R)$ is connected with the appearance of an imaginary part that vanishes exponentially as $R \to R_c$

As shown in [9], the cross section can be factorized if (1) is satisfied

$$\sigma(E, Z) = \sigma_0(Z) f(\eta), \qquad \eta = E/E_t. \tag{6}$$

The factor $f(\eta)$, which determines the energy dependence of the cross section of e^+ production, was calculated by us by substituting expressions (4) and (5) in formula (2.19) of [9] and by numerical integration (see Fig. 3). At the threshold $(\eta - 1) \ll 1$ we have:

$$f(\eta) = C(\eta-1)^{9/4}\exp\{-b(\eta-1)^{-1/2}\}, \tag{7}$$

where C and b are certain constants. from Fig. 3 we see that the use of the threshold asymptotic form (7) overestimates the cross section and can lead to noticeable errors at $\eta > 1.5$. At $\eta > 2$ the dependence of σ on E ceases to be strong, and the function $f(\eta)$ takes on values close to 10^{-3}. Further increase of the kinetic energy E does not lead to an appreciable growth of $f(\eta)$ and of the cross section σ, whereas the background effects increase [9]. It follows therefore that the region $E \simeq 2E$ is the optimal one for the performance of the experiment.

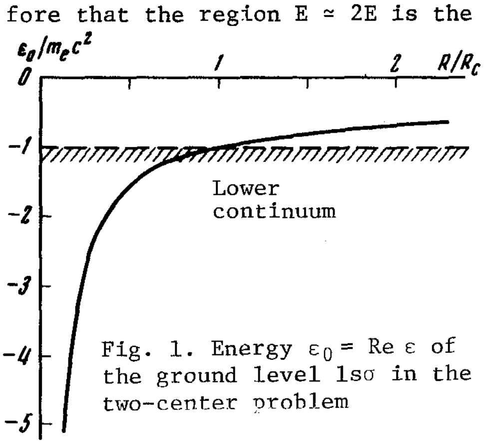

Fig. 1. Energy $\varepsilon_0 = \mathrm{Re}\ \varepsilon$ of the ground level $1s\sigma$ in the two-center problem

We present some typical figures. At Z = 95 we have $\sigma_0 \simeq \pi R_c^2 \sim 10^{-22}$ cm² and $\sigma \sim 10^{-25}$ cm² for $E > 2E_t = 1$ GeV. The relative velocity of the nuclei is then $v = 0.06\sqrt{\eta} \simeq 0.1$; this parameter determines the accuracy of the adiabatic approximation.

The analytic solution (2) - (5) of the two-center problem enables us to calculate not only the total cross section σ, but also the differential cross section $d\sigma/d\Omega$ of e^+ production at a fixed scattering angle θ, and also the energy spectrum of the produced positrons. Not being able to go into details, we shall describe these results qualitatively. As $E \to E_t$ the positrons can be observed only in backward scattering; with increase E, however, the range of suitable angles θ broadens rapidly and at $\eta > 2$ the maximum of $d\sigma/d\Omega$ begins to shift towards angles that are smaller than 180°. Registration of the scattered nuclei is desirable from the experimental point of view, since measurement of θ yields all the data for the nuclear trajectory, and lowers the background. As $E \to E_t$ the positron energy spectrum takes the form of a narroe peak near the maximum energy $T = T_m$ and broadens rapidly with increasing E, while the average energy $\bar{T} \ll T_m$. From (4) we get numerically that T_m is almost linear in E: $T_m \simeq 0.56(\eta - 1)m_e c^2$.

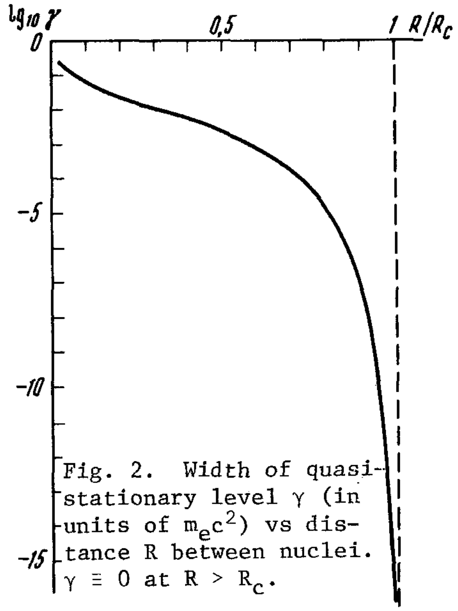

Fig. 2. Width of quasi-stationary level γ (in units of m_ec^2) vs distance R between nuclei. $\gamma \equiv 0$ at $R > R_c$.

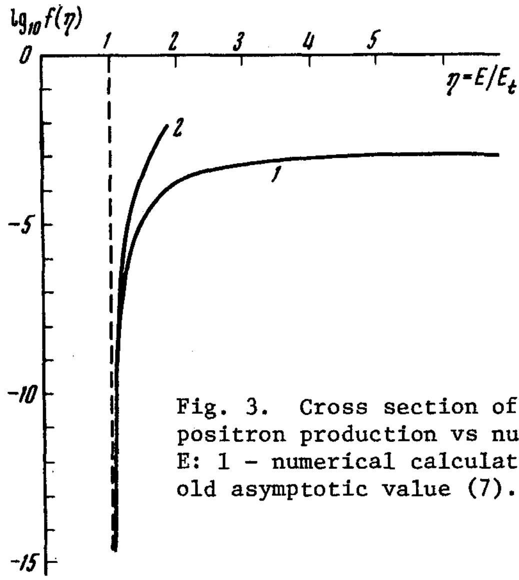

Fig. 3. Cross section of spontaneous positron production vs nuclear energy E: 1 - numerical calculation 2 - threshold asymptotic value (7).

Pair production from vacuum in a strong electric field is a characteristic prediction of quantum electordynamics, which has hitherto not been verified experimentally. Besides the effect considered above, this includes also pair production by a quasihomogeneous field $\vec{E}(t)$ that varies in time. The theory of this process has been recently the subject of many papers (see, e.g., [11] and the references cited therein). To produce one e^+e^- pair in the focus of a laser beam we must have $P \gtrsim 10^{19}$ W, which exceeds by several orders of magnitude the ratings of modern levels. It is therefore not excluded that spontaneous e^+ production in Coulomb collisions of nuclei will be more readily observable, since its cross section can reach $\sigma \sim 10^{-25}$ cm. Observation of this effect would verify Dirac's equation in strong external fields as well as the praperties attributed to vacuum in quantum field theory (see [5] for details).

The author thanks M. S. Marinov and I. S. Shapiro for a discussion of the work and I. M. Gokhfel'd for help with the calculations.

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1) Here $\hbar = c = m_e = 1$, E = incident-nucleus kinetic energy, E_t = positron spontaneous production threshold, the other symbols are from [9]. At $E = E_t$ the distance between the frontally-colliding nuclei is the critical $R = R_c$ at which the ground level of the electronic spectrum of the quasimolecule (Z_1, Z_2, e) drops to the limit of the lower continuum.

2) As $\varepsilon_0 \to -1$, the level width γ is exponentially small because of the Coulomb barrier for the slow positrons. If the level drops into the lower continuum by an amount $\sim m_ec^2$, then $\gamma \sim m_ec^2\exp(-4\pi Z\alpha) \simeq 100$ eV. The reason why γ is small in comparison with m_ec^2 can be understood with the aid of the effective-potential method developed in [4, 5].

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