$$n_2^* \approx J_\gamma n_2 \pi \lambda\!\!\!^{-2} \frac{f_2}{1+\alpha} F_\tau \, , \tag{3}$$

where f_2 is the probability of the Mossbauer effect in target 2. (For simplicity, we shall omit from now on the spin factors, and the level widths Γ_0, Γ_1, and Γ_2 will be assumed to be the same.) In (3) we have introduced a factor F_τ to allow for the inevitable time spreading of the two-stage excitation process as a result of the finite lifetime of the isomeric state in both targets. A trivial estimate shows that $(F_\tau)_{max} \simeq 0.2$.

Comparing (3) and (1), we find that the relevant density ratio of the excited nuclei in the two targets is given by

$$\frac{n_2^*}{n_1^*} \approx \beta F_\tau \frac{n_2}{n_1} \frac{\pi \lambda\!\!\!^{-2}}{\left\{\sigma_{n\gamma}^{(1)},\ \sigma_\gamma^{(1)}\right\}_{max}} \frac{f_1 f_2}{(1+\alpha)^2} \tag{4}$$

(it is assumed, naturally, that the cross section of the resonant nuclear γ-quantum absorption is large in comparison with the cross section for absorption by electrons).

Expression (4) has a very lucid structure. The feasibility of a relative increase in the excited-nucleus density in the second crystal is determined by the large multiplier $\pi\lambda\!\!\!^{-2}/\{\sigma_{n\gamma}^{(1)}, \sigma_\gamma^{(1)}\}_{max}$. This increase is partly offset by the last factor in (4), which depends on the ratio of the probabilities of the Mossbauer effect for the decay of the nucleus in the first crystal and absorption of the γ quanta in the second, and also on the decrease in the number of γ quanta emitted from the first crystal and their cross section for absorption in the second crystal by conversion ($(1+\alpha)^{-2}$). This factor makes it inconvenient to use two-stage pumping both for transitions with high energy (small values of f_1 and f_2), and for transitions with extremely small energy (large values of α). The optimal transitions are those with energies on the order of 30 - 50 keV, for which the ratio n_2^*/n_1^* can amount to 10 - 100.

[1] V. I. Gol'danskii and Yu. Kagan, Zh. Eksp. Teor. Fiz. 64, 90 (1973) [Sov. Phys.-JETP 37, No. 1 (1973)].

[2] V. I. Gol'danskii and Yu. Kagan, Paper at Science Session of the Division of General Physics and Astronomy and of the Division of Nuclear Physics, USSR Academy of Sciences, 28 December 1972. Usp. Fiz. Nauk 110, 445 (1973) [Sov. Phys. - Usp. 16, No. 4 (1974)].

ROLE OF DIFFRACTION CONTRIBUTIONS TO INELASTIC PROCESSES AT HIGH ENERGIES

I. D. Mandzhavidze
Physics Institute, Georgian Academy of Sciences
Submitted 22 May 1973
ZhETF Pis. Red. 18, No. 1, 64 - 67 (5 July 1973)

The limiting hadron multiplicity n_0 at which the hypothesis of the finite longitudinal correlation radius in momentum space is applicable is investigated from the point of view of the sum rules that relate the partial cross sections with the differential cross sections of "inclusive" processes. It is shown that such a picture of inelastic interactions can be valid in a region of multiplicity values that is broader than the region of the main maximum: $\bar{n} - n_0 > \sqrt{a \ln s/s_0}$. Thus, $n_0 = 3 - 4$ at an average multiplicity $\bar{n} = 10$.

1. It is known that in the description of inelastic hadronic processes one can neglect the diffraction contributions due to Pomeranchuk-pole exchanges at multiplicity values n close to the average $\bar{n} \simeq a \ln(s/s_0)$. With decreasing n, the pomeron exchanges become significant, since the relative energies of the secondary particles increase in this case. This paper is devoted to a determination of the region $n \geq n_0$ in which these exchanges can be neglected. This is done by deriving sum rules that connect the partial cross sections $\sigma_n(s)$ with the differential cross sections $d^3\sigma_n/dp^3$ of the "inclusive" processes. These sum rules are useful because they impose limitations on the amplitudes $A_n(p_a, p_b; p_1 \ldots, p_n)$ responsible for the detailed properties of the inelastic processes.

To derive the sought sum rules (the arguments presented below are analogous in many respects to those advanced in [1]), we introduce the partial cross sections

$$\sigma_n = \int d\tau_n |A_n|^2, \quad d\tau_n = (2\pi)^4 \delta^{(4)}(p_a + p_b - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3 p_i}{2(2\pi)^3 (p_i)_0}. \tag{1}$$

If we "lift" in (1) the integration with respect to the momentum of one of the particles of type c, then we obtain the differential cross section $d^3\sigma_n/dp^3$. By virtue of the symmetry of A_n with respect to permutations of identical particles and by virtue of the conservation law for the 4-momentum component, we obtain the sought sum rules by multiplying $d^3\sigma_n/dp_c^3$ by one of the momentum components $(p_c)_\mu$ and then integrating:

$$(p_a + p_b)_\mu \sigma_n = \int d^3 p_c (p_c)_\mu \frac{d^3\sigma_n}{dp_c^3}, \quad \mu = 0, 1, 2, 3, \tag{2}$$

where σ_n are defined in (1) and summation over all the types of particles, at the given n possible in the given reaction, is implied.

2. If the integration in (2) is carried out over a domain Ω_c smaller than the total phase volume, and we choose $\mu = 0$, then by virtue of the positiveness of the integrand we obtain the following inequality:

$$(p_a + p_b)_0 \sigma_n \geqslant \int_{\Omega_c} d^3 p_c (p_c)_0 d^3\sigma_n/dp_c^3 \tag{3}$$

which is more useful for a theoretical analysis. To reach this goal, the integration domain Ω_c must be chosen such that the Pomeranchuk contribution predomainate in the A_n of which $d^3\sigma_n/dp_c^3$ is made up. Then, retaining in σ_n only the non-diffraction contributions (denoted $\sigma_n^{(1)}$), we can determine from (3) the $n \geq n_0$ at which the diffraction contributions to σ_n can be neglected.

We determine Ω_c in the following manner: We choose s and $v = (p_a + p_b - p_c)^2$ to satisfy the conditions $s \gg v \gg s_0$. Alternately, introducing $v/s = 1 - x_c$ (where x_c has the meaning of the energy fraction caried away by particle c), then Ω_c in terms of x_c can be defined by

$$1 - x_c << 1. \tag{4}$$

In this case, confining ourselves to small t:

$$-t = -(p_a - p_c)^2 = \frac{\kappa^2 + m^2(1 - x_c)}{x_c} + 0(1/s) \sim s_0$$

$d^3\sigma_n/dp_c^3$ can be described with sufficient accuracy by the expression (it will be assumed henceforth that we have at our disposal only one type of particle):

$$\frac{d^3\sigma_n}{dp_c^3} = \frac{g^2(t)}{2(2\pi)^3 2(p_c)_0} (s/v)^{2\alpha(t)-1} B_{n-1}(v, t), \tag{5}$$

where $\alpha(t) = 1 + \alpha' t$ is the leading (Pomeranchuk) trajectory in the asymptotic form in s/v, and g(t) is the usual Regge vertex; the function $B_n(v, t)$ plays the role of the partial cross section for the production of n particles in the scattering of a vacuum reggeon by particle b. Using (5) and (3) we obtain after simple calculations

$$\sigma_n^{(1)}(\xi) \geqslant \frac{1}{16\pi^2} \int d|t| g^2(t) \int_{y_0}^{y_1} dy B_{n-1}(y,t) e^{2(\alpha(t)-1)(\xi - y)} \tag{6}$$

where $\xi = \ln(s/s_0)$, $y_i = \ln(v_i/s_0)$, and v_i is chosen such as to satisfy the condition (4). The areguments that follow now will be asymptotic in character, $\xi \gg y_1 \gg y_0$. Then the main dependence on t in (6) is determined by $\exp\{-2\alpha'|t|\xi$, since g(t) and $B_n(y, t)$ are regular functions of t in the physical region $t \leq 0$. This enables us to neglect the dependence of these functions on t, and inequality (6) takes the form

$$\sigma_n^{(1)} \geq \frac{\lambda}{\xi^{1+\gamma}} \int_{y_0}^{y_1} dy\, \sigma_{n-1}(y), \qquad s_0 \lambda = \frac{gr}{32\pi^2 \alpha'} . \tag{7}$$

In the derivation of this inequality we have assumed that in the form asymptotic in ξ

$$g B_n(y, t) = \Gamma(t) \sigma_n(y),$$

where, as shown in [2], the diffraction production of particles has a small value of the order of

$$s_0 \Gamma(t) = r(2\alpha' |t|)^\gamma .$$

We can therefore neglect the multipomeron exchanges in A_n, i.e., we can put in (7)

$$\sigma_n(y) = \sigma_n^{(1)}(y) . \tag{8}$$

3. To describe the nondiffractive part $\sigma_n^{(1)}$ we can use the most popular distribution, the Poisson formula

$$\sigma_n^{(1)}(\xi) = g^2 e^{-a\xi}(a\xi)^n / n! , \tag{9}$$

which describes the experimental data sufficiently well, and hopefully plays the major role when n is close to $\bar{n}$. Using (7), we can easily verify that neglect of the diffraction contributions to σ_n is not justified when $n \ll \bar{n}$. This can be seen by substituting (9) in (7) and taking (8) into account:

$$\frac{a\xi}{n} e^{-a\xi}(a\xi)^{n-1} \geq \frac{\lambda}{a\xi^{1+\gamma}} \int_{a y_0}^{a y_1} dy\, e^{-y} y^{n-1} , \tag{10}$$

and by choosing fixed n and y_i and letting $\xi \to \infty$; in this case the left-hand side decreases exponentially and the right-hand side only as a power of ξ.

To determine n_0, the integration limits y_i must be chosen such that the right-hand side of (10) is a maximum, but at the same time the condition (4), which guarantees the dominance of the Pomeranchuk contribution to $d^3\sigma_n/dp^3$, is satisfied. Namely:

$$y_i = \xi - \epsilon_i / a << \xi, \quad \epsilon_0 >> \epsilon_1 >> 1 .$$

The maximum of the integrand of (10) is then in the integration region. We find as a result that in the description of σ_n we can confine ourselves to the nondiffractive contributions (in our case, to the Poisson distribution) when

$$n \gtrsim n_0 = a\xi - \left(a\xi \ln \frac{a\xi^{1+\gamma}}{\lambda}\right)^{1/2} . \tag{11}$$

At contemporary energies, this range of n is broad enough. To estimate it we can use the correlation-function technique developed earlier in [3]. Using this technique, we can show that allowance for the correlation particle production distorts (11) insignificantly. Then Γ coincides with the three-pomeron vertex, the numerical value of which at low energies (assuming $\gamma = 0$) was obtained in [4]. Choosing total cross sections $\sigma = 40$ mb and $a \simeq 1$, we obtain $n_0 = 3 - 4$ (we recall that in our case $n \geq 2$) at an average multiplicity $\bar{n} = 10$.

In conclusion, I thank O. V. Kancheli for stimulating interest in the questions touched upon in the present article.

[1] C. E. De Tar, D. Z. Freedman, and G. Veneziano, Phys. Rev. D4, 906 (1971).
[2] R. C. Browez and J. H. Weis, Phys. Lett. 41B, 631 (1972).
[3] A. H. Mueller, Phys. Rev. D2, 2963 (1970).
[4] R. Rjaraman, Phys. Rev. Lett. 27, 693 (1971).