is known, however, that at the same temperatures and higher pressures such mixtures have second and third spontaneous-ignition pressure limits, between which such a gas mixture can exist quite well [3]. The conditions for the transition to these pressures are quite difficult to indicate, since there are no published quantitative experimental data on the ignition peninsular (with variation of the CS_2 and O_2 content). Another interesting possibility is that of obtaining the necessary combustion conditions in a compression shock wave.

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POSSIBLE EXISTENCE OF AUTOLOCALIZED STATES OF CARRIERS IN MAGNETIC SEMICONDUCTORS WITH NARROW BANDS

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It is shown in the present paper that autolocalized states can be thermodynamically favored in ferro- and paramagnetic semiconductors with $A \gg \Delta E \gg zIS^2$ (ΔE is the width of the band, $-2AS^{-1}\vec{s}\cdot\vec{S}_n$ and $2I\vec{S}_n\cdot\vec{S}_{n'}$ are the exchange energies of the electron interaction with the n-th and the neighboring atoms, z is the coordination number, and S is the spin of the atom).

It will be shown here that autolocalized fluctuon states, i.e., bound states of a carrier and magnetization fluctuation, can be thermodynamically favored in ferromagnetic and paramagnetic semiconductors with $A \gg \Delta E \gg zJS^2$ (ΔE is the width of the conduction band, $-2AS^{-1}\vec{s}\cdot\vec{S}_n$ and $-2J\vec{S}_n\cdot\vec{S}_{n'}$ are the exchange energies of the interaction of the electron with the n-th atoms and of the neighboring atoms with one another, z is the coordination number, and S is the spin of the atom).

Such large-radius states were investigated earlier for the opposite case $\Delta E \gg A \gg zJS^2$ [1 - 4]. On the other hand, in the case $A \gg \Delta E$, according to [5 - 7], the strong interaction leads to the formation of bound states of the electron and the atom spin. The resultant spin-electron complexes move in the crystals as unified quasiparticles with somewhat altered mass. At $T \neq 0$, according to [5], they remain in states of the band type, and the interaction with the spin deviations leads only to a raising of the bottom of the band by ΔE and to scattering. These results are indeed valid in the region of low temperatures. However, starting with a certain temperature T_1^*, autolocalization of the quasiparticle in a region of relatively large dimensions, with a strongly lowered spin disorder, is thermodynamically more favored. A decrease of its energy by ΔE can exceed the free-energy increase due to the formation of the magnetization inhomogeneity, and the autolocalized states are then stationary. Unlike the case $A \ll \Delta E$, it is not the electron but the spin-electron complex which becomes localized when the fluctuon is produced.

To investigate the possibility of fluctuon existence in the spin-wave region we recognize that at $2S \gg 1$ the band-edge shift due to the spin deviations is equal to [5] $\delta E = cd^2(2SN)^{-1}\sum_k k^2 n_k$, where $C = \Delta E(T = 0)(2z)^{-1}$, N is the number of cells, d is the length of the cubic-cell edge, and $n_{\vec{k}}$ are the spin-wave occupation numbers. In the case of a sufficiently smooth variation of the magnetization, we can introduce $\vec{r}$-dependent distributions of the numbers $n_{\vec{k}}$ in small volume elements, and regard δE as the potential energy of the quasiparticle. In the adiabatic approximation, its energy E_e is included as a term in the spin-excitation energy. The local spin-wave spectrum $\omega_{\vec{k}}(\vec{r}) = [2JS + C(2S)^{-1}v|\psi(r)|^2]d^2k^2$ (v is the cell volume) is determined not only by the direct interaction but also by the indirect one. By calculating in the usual manner the density of the free energy ϕ for this spectrum, we obtain the variation of the thermodynamic potential $\Delta\Phi$ min $I[\psi]$ as the quasiparticle goes over from the bottom of the band to an autolocalized state with wave function $\psi(r)$:

$$I[\psi] = \frac{\hbar^2}{2m}\int|\nabla\psi|^2 d\mathbf{r} - \frac{Cd^2}{2S}\frac{v}{8\pi^3}\int k^2 n_{\mathbf{k}}(\infty)\, d\mathbf{k} + \frac{kT}{8\pi^3}\int d\mathbf{r}\int d\mathbf{k}\left\{\ln\left[1-\exp\left(-\frac{\omega_{\mathbf{k}}(r)}{kT}\right)\right] - \ln\left[1-\exp\left(-\frac{\omega_{\mathbf{k}}(\infty)}{kT}\right)\right]\right\}. \qquad (1)$$

By determining min $I[\psi]$ by a direct variational method with a trial function $\psi_\alpha(\vec{r}) = (2\alpha/\pi)^{3/4}\exp(-\alpha r^2)$, we reduce the problem to a calculation of the minimum of the function $I(a)$, where

$$I(a) = \frac{\xi(5/2)}{\sqrt{2}\,\pi^2} C\nu^{2/3} S^{3/2}\left[Ka^{2/3} + \left(\frac{T}{\theta}\right)^{5/2}\left(\frac{f(a)}{a} - \frac{3\sqrt{\pi}}{4}\right)\right];$$

$$a = \frac{C\nu^{2/3}}{k\theta} v\left(\frac{2\alpha}{\pi}\right)^{3/2}; \quad \nu = \frac{d^3}{v}; \quad \theta = \frac{4JS\nu^{2/3}}{k}; \quad K = \frac{3\pi^3}{\sqrt{2}\,\xi(5/2)S^{3/2}}\mu^{2/3}; \qquad (2)$$

$$\mu = \frac{k\theta}{C\nu^{2/3}}; \qquad f(a) = \int_0^a \left(\ln\frac{a}{x}\right)^{3/2}(1+x)^{-5/2}dx.$$

At sufficiently small $K(\theta/T)^{5/2}$, the function $I(a)$ has, in addition to the minimum at $a = 0$ (i.e., at $\alpha = 0$) corresponding to the band states, also a second minimum at $a = a_0$, corresponding to the autolocalized states. When $T > T_1^*$ we have $\Delta\Phi = I(a_0) < 0$ and the fluctuon states are thermodynamically favored over the band states. It follows from numerical calculations that $T_1^* \simeq 7S^{-3/5}\mu^{4/15}\theta$, i.e., the fluctuons can be produced in the spin-wave region only at very small $\mu \lesssim 10^{-4}$ for ferromagnets with very low Curie temperature $T_C < 1°K$. At $T \sim T_1^*$ the effective number of atoms in the fluctuon is $n = v^{-1}|\psi(0)|^{-2} \sim \mu^{-1} >> 1$, and $-E_e \sim \mu^{-1/3}k\theta >> k\theta$, which justifies the macroscopic and adiabatic approximations.

Outside the spin-wave region of the ferromagnet, and in a paramagnet, the shift δE increases and fluctuons can be produced at much higher values, $T_C \sim$ 10 - 100°K. To investigate fluctuon formation in an ideal paramagnet, we use a simplified macroscopic model. We assume that outside a sphere of radius r_0 the spin state is the same as in the absence of an electron, and $\phi = \phi_{id} = -kTv^{-1}\ln(2S + 1)$, while the potential energy of the quasiparticle is equal to $\delta E = \kappa C$, where $1 < \kappa < z$. Inside the sphere, on the other hand, the state of the spins corresponds to a ferromagnet with the local spectrum $\omega_{\vec{k}}(\vec{r})$ given above (at $J = 0$). The value of r_0 is determined from the condition that the maximum frequency $\omega_{\vec{k}m}(r_0) = zCS^{-1}v\psi^2(r_0) = \kappa_k kT$ exceeds kT by several times ($\kappa_1 \gtrsim 2$).

By determining the minimum of the obtained functional $I[\psi]$ with the aid of the trial function $\psi_\alpha(r)$ and introducing in place of α a new variable $b = C(2SkT)^{-1}v(2\alpha/\pi)^{3/2}$, we obtain at $\ell = \ln(2zb/\kappa_1) >> 1$

$$\Delta\Phi = \min I(b);\; I(b)C^{-1} = \frac{3\pi}{2}\left(\frac{2\nu SkT}{C}b\right)^{2/3} - \kappa + \frac{2}{3\sqrt{\pi}}\frac{\ln(2S+1)}{S}\ell^{3/2}\frac{1}{b}. \qquad (3)$$

According to (3) we have in paramagnets $\Delta\Phi < 0$ and the fluctuons are thermodynamically favored at $T < T^*$, where we have in order of magnitude $kT^* \sim 0.03[\nu\ln(2S + 1)]^{-1}\ell^{3/2}\kappa^{5/2}C$. For example, at $2S = 3$, $2z/\kappa_1 = 6$, $\nu = 1$, and $\kappa = 3$ we have $kT^* \sim 0.1C$, i.e., $T^* \sim 100°K$ at $\Delta E \sim 1$ eV. $n \sim (\pi/\ell)^{9/10}(C/kT)^{3/5}$ and is large enough at $T << T^*$, but near T^* we have $n \sim 10$ and the employed macroscopic approximation yields only a very rough estimate.

A similar model can be used for a qualitative description of the fluctuons in ferromagnets outside the spin-wave region. It must only be recognized that here δE is determined by the interpolation formula $\delta E = \kappa'(T)C = \kappa_0'(1 - \eta)^p C$ (η is the relative magnetization, and $1 < p < 5/3$), and replace in (3) κ by $\kappa'(T)$ and $\ln(2S + 1)$ by $-v\phi(kT)^{-1}$. We then obtain for the lower temperature of transition to the fluctuon states the estimate $T_1^* \sim 0.03T(\nu v|\phi|)^{-1}\ell^{-3/2}\kappa_0'^{5/2}(1 - \eta)^{5p/3}C$. At $\eta < 1/3$ we have $kT_1^* \sim kT_C \sim (0.1 - 0.01)C$.

Thus, fluctuon states are thermodynamically favored at $A >> \Delta E$ and at sufficiently large $\Delta E/kT_C$ in a definite temperature interval $T_1^* < T < T^*$ that includes the Curie point, just as when $A << \Delta E$. The fluctuon-state characteristics, however, are essentially different from the case of broad bands; in particular, they do not depend on the value of A.

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ENERGY SPECTRUM OF ONE RANDOM ONE-DIMENSIONAL SYSTEM

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A one-dimensional model with random scattering is considered, in which the level densities have periodic singularities against the background of a continuous spectrum.

We consider a one-dimensional system describing the motion of a particle in an impurity field with potential energy in the form $U(x) = \sum_n U_n \delta(x - x_n)$. Following Lloyd [1], we shall assume that $x_n = \alpha n$, i.e., the scattering centers form a lattice, the amplitudes U_n are random quantities, there is no correlation between the distributions of U_n at different points of the lattice, and the probability density for a given n is

$$P(U_n) = \frac{1}{\pi} \frac{\gamma}{(U_n - U_0)^2 + \gamma^2} . \tag{1}$$

A unique distinguishing feature of such a model is that in spite of the random character of the scattering, the level density retains a certain "memory" of the lattice, mainfest by the presence in the level density of a periodic sequence of singularities against the background of a continuous spectrum.

The equation for the Green's function of the particle (ϵ is an infinitesimally small quantity)

$$\left[E + i\epsilon - \frac{\hat{p}^2}{2m} - \sum_n U_n \delta(x - x_n)\right] G(x, x') = \delta(x - x'). \tag{2}$$

Let $G(x, x')$ be the formal solution of (2) with $U_s = 0$. Elementary calculation leads to

$$G(x, x') = G_s(x, x') + U_s \frac{G_s(x, x_s) G_s(x_s, x')}{1 - U_s G_s(x, x')} . \tag{3}$$

We average $G(x, x')$ over U_s. It follows from (3) that G as a function of U_s has a simple pole in the upper half-plane ($\epsilon > 0$). Multiplying (3) by $P(U_s)$ and closing the integral with respect to U_s in the lower half-plane, we find that the result of the averaging is the replacement $U_s \to U_0 - i\gamma$. Repeating this reasoning for all the U_n, we find that the Green's function satisfies Eq. (2) with all $U_n = U_0 - i\gamma$. This makes it possible to express immediately the Green's function in terms of two linearly independent solution of the homogeneous equation (2), $\psi_{1,2}(x)$, for which the condition