ENERGY SPECTRUM OF ONE RANDOM ONE-DIMENSIONAL SYSTEM

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A one-dimensional model with random scattering is considered, in which the level densities have periodic singularities against the background of a continuous spectrum.

We consider a one-dimensional system describing the motion of a particle in an impurity field with potential energy in the form $U(x) = \sum_n U_n \delta(x - x_n)$. Following Lloyd [1], we shall assume that $x_n = \alpha n$, i.e., the scattering centers form a lattice, the amplitudes U_n are random quantities, there is no correlation between the distributions of U_n at different points of the lattice, and the probability density for a given n is

$$P(U_n) = \frac{1}{\pi} \frac{\gamma}{(U_n - U_o)^2 + \gamma^2} . \tag{1}$$

A unique distinguishing feature of such a model is that in spite of the random character of the scattering, the level density retains a certain "memory" of the lattice, mainfest by the presence in the level density of a periodic sequence of singularities against the background of a continuous spectrum.

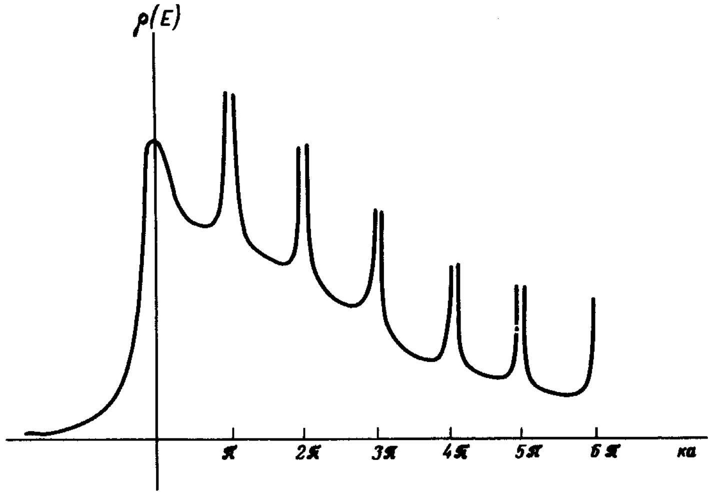
The equation for the Green's function of the particle (ϵ is an infinitesimally small quantity)

$$\left[E + i\epsilon - \frac{\hat{p}^2}{2m} - \sum_n U_n \delta(x - x_n)\right] G(x, x') = \delta(x - x'). \tag{2}$$

Let $G(x, x')$ be the formal solution of (2) with $U_s = 0$. Elementary calculation leads to

$$G(x, x') = G_s(x, x') + U_s \frac{G_s(x, x_s) G_s(x_s, x')}{1 - U_s G_s(x, x')} . \tag{3}$$

We average $G(x, x')$ over U_s. It follows from (3) that G as a function of U_s has a simple pole in the upper half-plane ($\epsilon > 0$). Multiplying (3) by $P(U_s)$ and closing the integral with respect to U_s in the lower half-plane, we find that the result of the averaging is the replacement $U_s \to U_0 - i\gamma$. Repeating this reasoning for all the U_n, we find that the Green's function satisfies Eq. (2) with all $U_n = U_0 - i\gamma$. This makes it possible to express immediately the Green's function in terms of two linearly independent solution of the homogeneous equation (2), $\psi_{1,2}(x)$, for which the condition



$$\psi_{1,2}(x+a) = e^{\pm i a}\,\psi_{1,2}(x)$$

is satisfied. The dispersion equation connecting the quasimomentum p with k ($k^2 = 2mE$, $h = 1$) is well known (see, e.g., [2])

$$\cos pa = \cos ka + \frac{k_0 - i\gamma}{k}\sin ka\,, \tag{4}$$

where

$$k_0 = mV_0\,, \quad \alpha = m\gamma\,.$$

A natural generalization of the formula for the level density in the one-dimensional case is

$$\rho(E) = \frac{1}{\pi}\,\mathrm{Re}\,\frac{dp}{dE}\,, \tag{5}$$

with $p = p' + ip''$. Formula (5) can be obtained also directly, by using the definition of ρ(E) in terms of the Green's function. The complex equation (4) for p has a solution for all values of k. By the same token we obtain the first result - the level density always differs from zero. The singular points for the level density are the values $k = k_n = n\pi/a$, where the term with iα vanishes in (4). An elementary calculation yields ($|k - k_n| \ll k_n$)

$$\rho(E) = \frac{mA_\pm}{2\pi k_n\sqrt{|k-k_n|}}\,, \qquad A_\pm = \left[\frac{\sqrt{k_0^2+\alpha^2} \mp k_0}{k_n a}\right]^{1/2}, \tag{6}$$

and the ± signs are determined by the signs of $k - k_n$. A general plot ρ(E) is shown schematically in the figure (for $k_0 > 0$).

Unfortunately, it is difficult to estimate the extent to which the result is sensitive to variations of the model.

In conclusion, I am grateful to A.M. Dykhne for calling my attention to Lloyd's model.

[1] I.C. Lloyd, Solid State Phys. 2, 10 (1969).
[2] C. Kittel, Introduction to Solid State Physics, Wiley, 1961.

DECAY OF INITIAL DISCONTINUITY IN THE KORTEWEG-DE VRIES EQUATION

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As is well known, processes in nondissipative media with small nonlinearity and dispersion are described by the Korteweg-deVries equation

$$\frac{\partial \eta}{\partial t} + \eta \frac{\partial \eta}{\partial x} + \frac{\partial^3 \eta}{\partial x^3} = 0, \tag{1}$$

So far, however, (1) was used to investigate mainly the evolution of a perturbation that is concentrated at the initial instant in a finite region of space [1 - 3]. The purpose of the present paper is to solve a problem in which η experiences at the point $x = 0$ a finite jump, so that $\eta = \eta_0$ at $x < 0$ and $\eta = 0$ at $x > 0$. In the course of time this discontinuity changes into a broadening region occupied by the oscillations. At $\eta_0^{3/2} t \gg 1$, the dimension of this region is much larger than the oscillation wavelength, so that Witham's quasi-classical method can be used [4]. Equation (1) has a periodic solution

$$\eta(x,t) = \frac{2a}{s^2}\, dn^2\left[\left(\frac{a}{6s^2}\right)^{1/2}(x - Vt), s\right] + \gamma$$
$$V = \frac{2a}{3s^2}(2 - s^2) + \gamma, \tag{2}$$

where $dn(u, s)$ is a Jacobi elliptic function with modulus s, $0 \le s \le 1$. The value $\bar{\eta}$ averaged over the period and the wave vector k are given by

$$\bar{\eta} = \gamma + \frac{2a\,E(s)}{s^2 K(s)}, \qquad k = \frac{\pi}{K(s)}\left(\frac{a}{6s^2}\right)^{1/2} \tag{3}$$

where K and E are complete elliptic integrals of the first and second kind, respectively.

We seek a solution in the form (2), assuming a, s, and γ to be slowly-varying functions of x and t. It is convenient to write the approximate equations for these functions, according to [4], by introducing three new quantities $r_3 > r_2 > r_1$:

$$r_2 - r_1 = 2a, \quad \frac{r_2 - r_1}{r_3 - r_1} = s^2, \quad r_1 + r_2 - r_3 = 2\gamma.$$

The equations for r_α are

$$\frac{\partial r_\alpha}{\partial t} + v_\alpha \frac{\partial r_\alpha}{\partial x} = 0, \qquad \alpha = 1, 2, 3; \tag{4}$$

where v_α are definite functions of r_α; we need only an expression for v_2: