$$\ell_w = \sqrt{G} \sim 0.66\cdot 10^{-16} \text{ cm}.$$

In conclusion, the authors are sincerely grateful to A.P. Rekalo for useful discussions.

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INFLUENCE OF LOW-FREQUENCY OSCILLATIONS ON TRANSPORT PROCESSES ACROSS A MAGNETIC FIELD

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The author has shown in an earlier paper [1] (see also [2, 3]) that the presence of low oscillations of the potential in a plasma can lead to a noticeable increase of the transport coefficients across a strong magnetic field. This is connected with the "mixing" [4] that results from the deviation of the drift trajectories of the particles from magnetic surfaces (or from constant-pressure surfaces).

The cited papers, however, considered only a two-dimensional case, when the perturbation potential does not depend on one of the coordinates[1]. In the present article we get rid of this limitation and analyze the influence exerted on the transport across a strong magnetic field by low-frequency three-dimensional potential oscillations (e.g., drift waves). We confine ourselves here to a qualitative theory, which explains easily the physical picture of the phenomena in question, on one hand, and gives sufficiently satisfactory quantitative description of the process, on the other.

We consider a plasma cylinder situated in a longitudinal (B_z) and azimuthal ($B_\phi = \theta B_z$) magnetic fields, in which there propagates a potential wave of the form

$$\Phi = \Phi_0(r, \eta - v_0 t)\exp i[k_\perp \eta + k_\parallel \zeta - \omega' t], \quad (1)$$

where $\zeta = z + \theta r\phi$ is the coordinate along the force line, $\eta = r\phi - \theta z$ is the transverse coordinate, $\omega' = \omega + k_\perp v_0$, $v_0 = -c(E_r/B)$ is the velocity of the azimuthal rotation of the plasma in a constant (ambipolar) radial electric field E_r, and ω is the frequency of the oscillations in a coordinate system in which the plasma is at rest[2]. We assume that the magnetic field is strong enough so

[1]Thus, in [1, 2] the potential was independent of the longitudinal coordinate of the torus, and in [3] it was constant along the line $k_y y + k_z z$ = const.

[2]To abbreviate the notation, we assume here that $\theta \ll 1$.

that the particle motion can be described by the drift equations. Then, since the particles (electrons or ions), move along the force line with velocity $\dot{\zeta} = u$ and drift transversely to it with velocity $\dot{\eta} = v_0$ in the absence of oscillations of the potential, it is easily seen that the particles moving with velocities in the interval

$$\frac{\omega}{k_\parallel} - \Delta u_j < u < \frac{\omega}{k_\parallel} + \Delta u_j$$

where

$$\Delta u_j = v_j\left(\frac{2e_j\Phi_0}{T_j}\right)^{1/2}, \quad v_j = \left(\frac{T_j}{m_j}\right)^{1/2}, \text{ and } e_j,\ m_j,\ T_j$$

and e_j, m_j, and T_j are the charge, mass, and temperature), are trapped by the wave and drift under the influence of the wave field transversely to the force line with velocity $v_E \simeq (\partial\Phi_0/\partial\eta)(c/B)$ over the surface $\Phi_0(r, \eta) = \text{const}$[3]. In other words, the particles trapped by the wave are deflected from the magnetic surface by an amount $a \simeq (\partial \ln\Phi_0/\partial\eta)^{-1}$. Thus, if the effective collision frequency $\nu_j^* = \nu_j(v_j/\Delta u_j)$ is smaller than the characteristic frequency $\nu_0 = v_E/a$ of the particle motion in the wave field, then as a result of each collision, which transforms the particle from a trapped one into an untrapped one, the particle will be displaced on the average by an amount $\nu_j \simeq a$[4]. In the opposite case of large collision frequencies, $\nu_j^* \gg \nu_0$, the particle moves during the time between collisions a distance smaller by a factor ν_j^*/ν_0, and consequently the effective displacement is $\Delta_j \simeq a(\nu_0/\nu_j^*)$. Combining these two formulas and recognizing that the relative number of trapped particles is $\Delta N_j/N = (\Delta u_j/u_j)\exp(-\omega^2/2k_\parallel^2 v_j{}^2)$, we obtain for the diffusion (thermal conductivity) coefficient connected with the trapped particles the following expression

$$D_{tr} \simeq \nu_j^* \Delta_j^2 \frac{\Delta N_j}{N} = \frac{\nu_j v_j}{\Delta u_j} \frac{a^2 \exp\left(-\omega^2/2k_\parallel^2 v_j^2\right)}{1 + \nu_j^{*2}/\nu_0^2}. \tag{2}$$

At a sufficiently high collision frequency, it becomes necessary to take into account also the contribution made by the untrapped particles, whose velocity is close to the phase velocity of the wave and which are consequently strongly displaced in the field of this wave. At $k_\parallel \Delta u_j \Delta u_j^2/v_j^2 < \nu_j < k_\parallel v_j$, the rms displacement of the untrapped particles during the time between two

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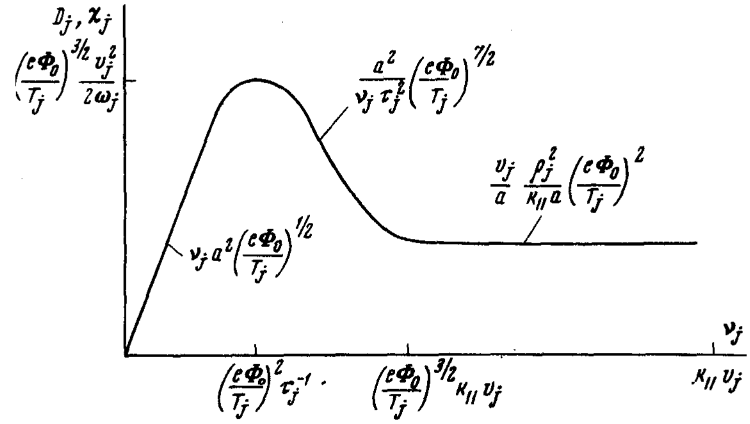
[3]It should be noted that the particle is trapped by the wave if the relative change $\Delta B/B$ of the magnetic field along the force line over the wavelength does not exceed the depth $e\Phi_0/T_j$ of the potential well. For toroidal particles of the Tokamak type, at $k_\parallel = \theta/r$ and $\Delta B/B = r/R$ this condition takes the form $r/R < e\Phi_0/T_j$.

[4]The factor $v_j^2/\Delta u_j^2$ in the effective collision frequency is the result of the differential character of the integral of the Coulomb collisions ($st_j \sim \nu_j v_j^2 (\partial^2 f_j/\partial u^2)$) and should be left out if ν_j is taken to mean the frequency of collisions with neutral particles.



Diffusion coefficient D_j (thermal-conductivity coefficient χ_j) vs. the collision frequency ν_j at $(e\Phi_0/T_j)^{1/2} < \tau_j k_\| v_j$ and $k_\perp a \leq 1$.

successive collision will obviously be of the order of $\Delta_j^2 \simeq v_E^2/[(\omega - k_\| u_j)^2 + \nu_j^2]$, and consequently the contribution made to the diffusion (thermal conductivity) coefficient by the untrapped particles is[5]

$$D_{utr} \approx \frac{v_E^2}{k_\| v_i} \frac{\exp\left(-\frac{\omega^2}{2k_\|^2 v_i^2}\right)}{\left[1 + k_\|^2 \Delta u_i^6 / v_i^4 \nu_i^2\right]^{1/2}}. \quad (3)$$

Summing (2) and (3), we obtain finally an expression for the diffusion and thermal conductivity coefficients; this expression is valid in the region of sufficiently small collision frequencies[6] $\nu_j < k_\| v_j$

$$D_i \approx \chi_i \approx \nu_i \left\{ \frac{\left(\frac{e_i\Phi_0}{T_i}\right)^{-1/2} a^2}{1 + (\nu_i \tau_i)^2 \left(\frac{e_i\Phi_0}{T_i}\right)^{-4}} + \frac{a^2\left(\frac{e_i\Phi_0}{T_i}\right)^2 (\tau_i^2 \nu_i k_\| v_i)^{-1}}{\left[1 + \left(\frac{e_i\Phi_0}{T_i}\right)^3 \left(\frac{k_\| v_i}{\nu_i}\right)^2\right]^{1/2}} \right\} \exp\left(-\frac{\omega^2}{2k_\|^2 v_i^2}\right), \quad (4)$$

where $\tau_j = a^2/\rho_j v_j$, $\rho_j = v_j/\omega_j$, and $\omega_j = e_j B/m_j c$.

We note that expression (4) is valid only if the particle can drift between two successive reflections over a distance much shorter than a, i.e., at $v_E \ll k_\| a \Delta u_j$ or at $|e\Phi_0/T_j| \ll k_\| v_j \tau_j$). For electrons this condition is practically always satisfied, whereas for ions it may be violated at sufficiently small $k_\|$. In this case the particles trapped by the wave play practically no role, and the diffusion (thermal conductivity) coefficient is determined entirely by the

[5]We have taken into account here the fact that at low collision frequencies, $\nu_j^* = \nu_j v_j^2/\Delta u_j^2 \ll k_\| \Delta u_j$, the displacement is of the order of $\Delta_j \simeq v_E/k_\| \Delta u_j$.

[6]We note that the ambipolar diffusion coefficient can be smaller than that given by formula (4), since it is determined by the smaller of the electron or ion diffusion coefficients.

untrapped particles and can be easily estimated. It turns out to equal[7)]

$$D_i \approx \chi_i \approx \left(\frac{e_i\Phi_o}{T_i}\right)^2 \frac{a^2}{r_i^2 k_{\parallel} v_i} \frac{\exp\left(-\omega^2/2k_{\parallel}^2 v_i^2\right)}{\left[1+\left(\frac{e_i\Phi_o}{T_i}\right)^6 (\nu_i r_i)^{-2}(r_i k_{\parallel} v_i)^{-4}\right]^{1/2}}. \tag{5}$$

It follows from (4) and (5) that the presence of oscillations of the type (1) in the plasma can lead to an appreciable increase of both the diffusion and the thermal conductivity in comparison with their values given by the neoclassical theory (see the figure).

We note in conclusion that in the case of drift waves, since their longitudinal phase velocity $\omega/k_{\parallel}$ is usually much higher than the thermal velocity of the ions (but is lower than the thermal velocity of the electrons), the presence of such oscillations in a plasma may also not lead to a noticeable increase of the transport coefficients for the ions (owing to the presence of the small factor $\exp(-\omega^2/2k_{\parallel}^2 v_j^2)$).

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THRESHOLD OF DECAY INSTABILITY IN AN INHOMOGENEOUS PLASMA

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The purpose of this communication is to point out that in a spatially inhomogeneous plasma there can be no absolute stability of the decay type with an excitation threshold lower than given by the recent results [1 - 4]. Such an instability arises if both parametrically-coupled waves have turning points inside the plasma layer, as shown in the figure. The resonant interaction is realized in the region of intersection of the curve (the pump field is assumed to be homogeneous); it leads not only to an amplification of the incident wave, but also to the appearance of a wave of another type [2, 4]. It is easy to verify that if the energy propagates as indicated in the figure, then such a nonlinear transformation produces positive feedback and can lead to a growth of the wave amplitudes with time. If the energy of one of the waves is transported in the opposite direction, then there is no feedback, and we have only amplification, as before. The situation described here is usual in an inhomogeneous plasma and can be realized for a large number of different decay processes. In the absence of damping, the instability sets in if the product of the coefficients of the transformations 1 → 2 and 2 → 1 exceeds unity. According to

[7)] In the derivation of formulas (4) - (5) we have assumed that $k_{\perp}a \ll 1$, and formula (5) is valid only at $(k_{\perp}a)^{-1} > (v_E/k_{\parallel}a\Delta u_j) > 1$. If $k_{\perp}a \gtrsim 1$, then there is no region where (5) is valid, and the diffusion (thermal conductivity) coefficient is determined by formula (4), where a in the second term should be replaced by $k_{\perp}^{-1}$.