untrapped particles and can be easily estimated. It turns out to equal[7])

$$D_i \approx \chi_i \approx \left(\frac{e_i\Phi_o}{T_i}\right)^2 \frac{a^2}{r_i^2 k_\parallel v_i} \frac{\exp\left(-\omega^2/2k_\parallel^2 v_i^2\right)}{\left[1+\left(\frac{e_i\Phi_o}{T_i}\right)^6 (\nu_i r_i)^{-2} (r_i k_\parallel v_i)^{-4}\right]^{1/2}} . \quad (5)$$

It follows from (4) and (5) that the presence of oscillations of the type (1) in the plasma can lead to an appreciable increase of both the diffusion and the thermal conductivity in comparison with their values given by the neoclassical theory (see the figure).

We note in conclusion that in the case of drift waves, since their longitudinal phase velocity $\omega/k_\parallel$ is usually much higher than the thermal velocity of the ions (but is lower than the thermal velocity of the electrons), the presence of such oscillations in a plasma may also not lead to a noticeable increase of the transport coefficients for the ions (owing to the presence of the small factor $\exp(-\omega^2/2k_\parallel^2 v_j^2)$).

[1] L.M. Kovrizhnykh, ZhETF Pis. Red. 13, 513 (1971) [JETP Lett. 13, 365 (1971)].
[2] L.M. Kovrizhnykh, Zh. Eksp. Teor. Fiz. 62, 1345 (1972) [Sov. Phys.-JETP 65, 709 (1972)].
[3] O.P. Pogutse, Nuclear Fusion 11, No. 1 (1972).
[4] G.I. Budker, in: Fizika plazmy i problema upravleniya termoyadernykh reaktsii (Plasma Physics and the Problem of Controlled Thermonuclear Reactions), Vol. 1, AN SSSR, 1958, p. 66.

THRESHOLD OF DECAY INSTABILITY IN AN INHOMOGENEOUS PLASMA

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The purpose of this communication is to point out that in a spatially inhomogeneous plasma there can be no absolute stability of the decay type with an excitation threshold lower than given by the recent results [1 - 4]. Such an instability arises if both parametrically-coupled waves have turning points inside the plasma layer, as shown in the figure. The resonant interaction is realized in the region of intersection of the curve (the pump field is assumed to be homogeneous); it leads not only to an amplification of the incident wave, but also to the appearance of a wave of another type [2, 4]. It is easy to verify that if the energy propagates as indicated in the figure, then such a nonlinear transformation produces positive feedback and can lead to a growth of the wave amplitudes with time. If the energy of one of the waves is transported in the opposite direction, then there is no feedback, and we have only amplification, as before. The situation described here is usual in an inhomogeneous plasma and can be realized for a large number of different decay processes. In the absence of damping, the instability sets in if the product of the coefficients of the transformations 1 → 2 and 2 → 1 exceeds unity. According to

[7])In the derivation of formulas (4) - (5) we have assumed that $k_\perp a \ll 1$, and formula (5) is valid only at $(k_\perp a)^{-1} > (v_E/k_\parallel a\Delta u_j) > 1$. If $k_\perp a \gtrsim 1$, then there is no region where (5) is valid, and the diffusion (thermal conductivity) coefficient is determined by formula (4), where a in the second term should be replaced by $k_\perp^{-1}$.

[4], this condition is of the form

$$2 \lesssim e^{2\pi z} , \qquad (1)$$

where $z = |\gamma^2 \ell^2 / u_1 u_2|$, γ is the increment of the decay instability of the waves under consideration in a homogeneous plasma, u_i is the x component of the group velocity, and $\ell^{-2} = |\partial(k_{1x} - k_{2x})/\partial x|_{k_1 - k_2}$. The right-hand side of (1) is the energy gain of the wave in the resonant region. In the absence of at least one turning point, when no instability sets in, the decay interaction can be conditionally regarded as dangerous if this coefficient exceeds a certain large value, e.g., e^{10} [1]. Since $z \sim E_0^2$ (E_0 is the amplitude of the pump wave), the threshold value of E_0 in the presence of feedback is decreased by an approximate factor of 3. In addition to (1), it is necessary also to satisfy the relation $\Delta\phi = 2\pi m$, where $\Delta\phi$ is the total change in the phase of the wave on going over the closed path joining the turning points and the resonance points. In the vicinity of the turning points, when $k_{xi} = \sqrt{|\alpha_i(x - x_i)|}$, where α_i and x_i are constants, it takes the form

$$\frac{2}{3}\beta^{3/2} + \delta = \pi m, \quad m = 1, 2, 3\ldots, \qquad (2)$$

where $\beta = |(x_1 - x_2)/a|$, $a = (1/\alpha_1 - 1/\alpha_2)$, and $\delta(z) \lesssim 1$. In the same approximation,

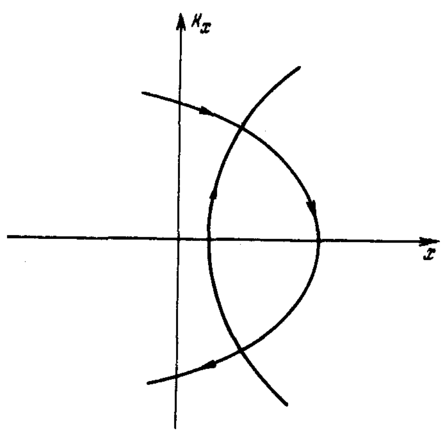
$$z = \frac{\gamma^2 x_1' x_2'}{8a^2\sqrt{\beta}} , \qquad x_i' = \left| \frac{dx_i}{d\omega_i} \right| .$$

In the presence of weak damping, the left-hand side of (1) should be replaced by $1 + \exp(2\sqrt{\beta\beta''})$, where $\beta'' = \nu_1 x_1' + \nu_2 x_2'$ and ν_i is the linear damping decrement, while Eq. (2) remains in force. It follows from it that there is an infinite set of unstable modes whose frequencies are given by the condition

$$\beta = \alpha_m^2 , \qquad \alpha_m = \left(\frac{3}{2}\pi m\right)^{1/3}$$

(and by the decay condition $\omega_1 + \omega_2 = \omega_0$). For the instability threshold of the m-th mode we have

$$\gamma^2 = \frac{4\alpha_m a^2}{\pi x_1' x_2'} \ln[1 + \exp(2\alpha_m \beta'')] . \qquad (3)$$



The exact solution of the equations describing the coupled waves confirms the foregoing relations; it follows also from this solution that in the case of strong damping, $\beta'' \gg \alpha_m^2$, the instability threshold is given by

$$\gamma^2 \approx \frac{(\beta'')^2 a^2}{x_1' x_2'} . \qquad (4)$$

By way of illustration, we consider the excitation of Langmuir oscillations and ion sound in an isotropic nonisothermal plasma by an electromagnetic wave. In this case $x_1' = 2\ell_n/\omega_\ell$, $x_2' = 2\ell_T/\omega_s$, $a = (3r_D^2\ell_n + k_1^{-2}\ell_T)^{1/3}$, $\ell_n^{-1} = d(\ln n_e)/dx$, $\ell_T^{-1} = d(\ln T_e)/dx$, k_1 is the projection of the wave vector on a direction perpendicular to the inhomogeneity, $\omega_\ell^2 = 4\pi ne^2/m_e$, $\omega_s^2 = T_e k_\perp^2/m_i$ and $r_D^2 = T_e/4\pi n$. Assuming that $T_e >> T_i$ and $k_\perp^{-2}\ell_T >> r_D^2\ell_n$, and taking into account only the Landau damping for ion sound, we have $\beta'' = (\pi/2)^{1/2}(m_e/m_i)^{1/2}(k_\perp\ell_T)^{2/3}$. Using the known value for γ^2

$$\gamma^2 = \frac{1}{4}\left(\frac{E_o}{4\pi n T_e}\right)^2 \omega_\ell \omega_s$$

we see that the most stable are short-wave oscillations. If the inhomogeneity of T_e is not very large, $\ell_T >> r_D(m_i/m_e)^{3/4}$, then at $k_\perp \lesssim 1/r_D$ we obtain from (4) if m = 1

$$\left(\frac{E_o}{4\pi n T_e}\right)^2 \approx \left(\frac{m_e}{m_i}\right)\left(\frac{\ell_T}{\ell_n}\right).$$

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[1] F.W. Perkins and I. Flick, Phys. Fluids 14, 2012 (1971).
[2] A.D. Piliya, Tenth International Conference on Phenomena in Ionized Gases, Paper No. 4.3.8.1, Oxford, England, 1971.
[3] M.N. Rosenbluth, Phys. Rev. Lett. 29, 565 (1972).
[4] A.D. Piliya, Zh. Eksp. Teor. Fiz. 64, No. 4 (1973) [Sov. Phys.-JETP 37, No. 4 (1973)].

GAS-LIQUID PHASE TRANSITION IN THE SYSTEM OF SURFACE IMPURITIES OF ^{3}He IN SUPERFLUID SOLUTIONS

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It is shown that a first-order phase transition should be observed on the free surface of an He^3 solution in helium II. This transition is due to condensation of the surface impurities and is accompanied by a kink on the plot of the surface tension against the temperature.

Surface impurity levels of He^3 exist on the free surface of a solution of He^3 in He^4 [1 - 4]. At high temperatures the impurities should behave like a classical two-dimensional ideal gas, as is confirmed by the experiments of Zinov'eva and Boldarev [2]. At low temperatures, as shown by Edwards et al. [3], the surface impurities behave as a Fermi liquid. The authors of [3] have processed their own data under the assumption that at low temperatures the system behaves like a rarefied Fermi gas. This assumption was based on the fact that the experimental dependence of the contribution of the impurities to the surface tension depends quadratically on the chemical potential μ as $T \to 0$, as is typical of a rarefied Fermi gas. It turns out then that the temperature dependence of the surface tension is the same as in a strongly degenerate Fermi system, even at temperatures greatly exceeding the degeneracy temperature of the rarefied gas.

A possible explanation of this fact is that at low temperatures the surface impurities actually form a dense two-dimensional liquid rather than a rarefied gas. In this case the dependence of the impurity part of the surface