COUPLING CONSTANT AND FORM FACTOR FOR THE $t \rightleftarrows d + n$ VERTEX

Yu.V. Orlov and V.B. Belyaev
Nuclear Physics Institute of the Moscow State University
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It has been observed that the coupling constant G_t^2 for the $t \rightleftarrows d + n$ vertex is very sensitive to the form of the NN potential; this can be used to select NN interactions that agree with the results of the analysis of nuclear reactions and scattering by light nuclei.

Calculations of the form factor W and an estimate of the coupling constant G_t^2 for the $t \rightleftarrows d + n$ vertex were performed in the present paper for two central potentials with soft repulsion core, those of Malfliet and Tjon (MT) [1] and of Darewich and Green (DG) [2], acting in the $^3S_1$ and in the $^1S_0$ states and describing the corresponding scattering phase shifts in the energy interval from 0 to 300 - 400 MeV. W was calculated from the formulas of [3], which were generalized to take into account the spin and the isospin (see also [4 - 6]). Only the contribution of the S-wave was taken into account. We used the tritium wave functions v and u (their definition is given, e.g., in [7]), obtained by solving the Faddeev equations by the Bateman method [8, 9]. Figure 1 shows the results of the calculation of W for the case of a real deuteron ($W = W_1$) and a real neutron ($W = W_2$). We see that the form factors, particularly W_2, are quite sensitive to the potential. Simultaneous emergence of the deuteron and neutron to the mass shell corresponds to taking the limit as $Q^2 \to -\kappa^2$, where $Q^2 = (p_d - 2p_n)^2/9$, $\vec{p}_i$ is the momentum of particle i, $\kappa^2 = (4m/3)(\varepsilon_t - \varepsilon_d)$, m is the mass of the nucleon, and ε_t and ε_d are the binding energies of tritium and the deuteron (we use a system of units in which $\hbar = c = 1$). At this point $W_1 = W_2 = G_t$. The functions v and u were calculated only for $Q^2 > 0$. Extrapolation of W_1 or W_2 to the point $Q^2 = -\kappa^2$ is difficult, since it is precisely here where the form factors vary most rapidly. A more convenient procedure is proposed in [10] and makes use of the fact that the function v(q, Q) has a pole at $Q^2 = -\kappa^2$, and the residue at this pole is proportional to $G_t\phi(q)$, where $\phi(q)$

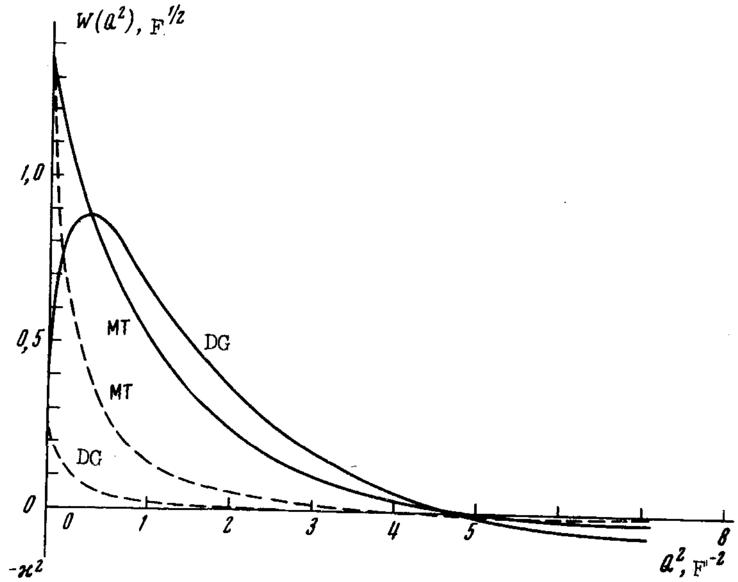

Fig. 1. The form factors W_1 (solid) and W_2 (dashed) for Malfliet-Tjon (MT) and Darewich-Green (DG) potentials. W_1 corresponds to a real deuteron and a virtual neutron, and W_2 to a virtual deuteron and real neutron. The curves in the region $-\kappa^2 \le Q^2 \le 0$ were obtained by extrapolation.

is the spatial wave function of the deuteron in the momentum representation (the function $\phi(q)$ was calculated also by the Bateman method). This leads to the relation

$$G_t = \lim_{Q^2 \to -\kappa^2} G(q, Q), \tag{1}$$

$$G(q, Q) = -(3\sqrt{3}/4m)(Q^2 + \kappa^2)\, v(q, Q)/\phi(q).$$

It is convenient to extrapolate in accordance with formula (1), for at sufficiently large q the function G(q, Q) varies almost linearly with Q^2, with a small derivative (see Fig. 2). This is natural, since large values of q correspond to a more compact deuteron cluster, i.e., the asymptotic value with respect to the variable ρ, which is conjugate to the momentum Q, is realized at shorter distances. This is tantamount to saying that the pole term in the function v(q, Q) is dominant even in the physical region of the variable Q^2 at sufficiently small $Q^2 (> 0)$. Extrapolation by means of formula (1) yields the coupling constants $G_t^2(\mathrm{MT}) \simeq 1.9$ F and $G_t^2(\mathrm{DG}) \simeq 0.1$ F, which differ by a factor ∿20 (the result of [1] for a Reid potential with soft core corresponds to[1]) $G_t^2 \simeq 2$ F). We note that the three-nucleon characteristics, which unlike G_t (see (1)) have an integral character, do not differ very strongly for the MT and DG potentials (see Table VI of [9]). Let us see what differences between the potentials lead to such a strong effect in G_t^2. We call attention first to the correlation existing between the ratio of the triplet potential $V_t(r)$ to the singlet potential $V_s(r)$ and of the functions v(q, Q) and u(q, Q). We recall that when $V_t(r) = V_s(r)$ we have v = u (the spatial part of the wave function is fully symmetrical with respect to nucleon permutation). In the case of the MT potential, V_t and V_s differ only in depth, with $|V_t(r)| > |V_s(r)|$. It follows furthermore from the calculations that v > u (all the statements made here and below pertain only to the region of interest to us, that of sufficiently small Q^2). It is therefore natural to assume that v < u if $|V_t| < |V_s|$. In the case of the DG potential, calculation yields v < u in the entire region of Q (at fixed q) of importance for the normalization integral (the relative contributions of u and v to the normalization, which shows also the admixture $P_{s'}$ of states

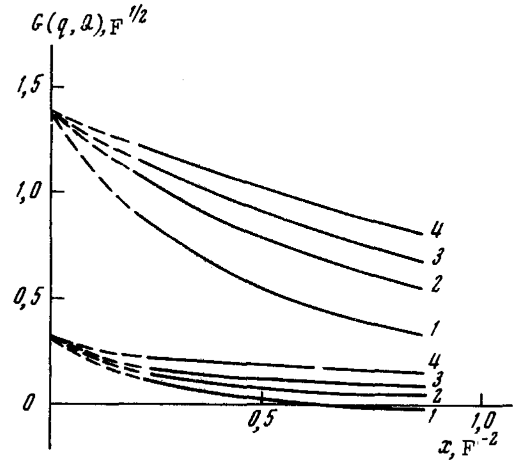

Fig. 2. Dependence of G(q, Q) on $x = Q^2 + \kappa^2$ at different q. The upper bundle of curves was obtained for the MT potential at the following values of q (F^{-1}): 1 - 0.597, 2 - 1.005, 3 - 1.585, 4 - 3.127; the lower bundle was obtained for the DG potential with the following values of q: 1 - 1.210, 2 - 0.456, 3 - 2.806, 4 - 2.651. The dashed line shows extrapolation to the value $Q^2 = -\kappa^2$.

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1) In [10] they determined the dimensionless quantity $C = A_0/\sqrt{2\kappa}$ (A_0 is the coefficient in the asymptotic wave function), which is connected with G_t by the relation $G_t^2 = (9/2)\pi ƛ \kappa N C^2$, where $ƛ = \hbar/mc$ and N (=3) is a factor that takes the identity of the nucleons into account. The result ($C^2 = 2.8$) [10] was obtained for the $h \rightleftarrows p + d$ vertex but $G_h^2 = G_t^2$ accurate to the extent of the deviation from charge independence.

of mixed symmetry). The difference between the functions v for the MT and DG potentials is seen also directly in Fig. 2, since the deuteron functions practically coincide for both potentials. The visible decrease of v leads to a decrease of G_t^2, i.e., to a decrease in the weight of the cluster state (d + n) in tritium in the case of the DG potential. For this potential, the difference between $V_t(r)$ and $V_s(r)$ has a more complicated character. In the region $r \lesssim 1.85$ F we have $|V_t| > |V_s|$, and the ratio V_t/V_s reaches ~ 2 (for the MT potential we have $V_t/V_s \simeq 1.2$). If this region were to play an important role in the values of u and v at small Q^2, then the inequality $v > u$ (which holds for the MT potential) would only become stronger for the DG potential. The inverse inequality ($v_{DG} \ll u_{DG}$) can only mean, if our foregoing assumption is correct, that the ratio of these functions is determined by the ratio of the potentials V_t and V_s in the peripheral region precisely where $|V_t^{DG}| < |V_s^{DG}|$. We note also that in the external region the DT potential tends to zero much more rapidly than the MT potential (the arguments in the exponential differ by a factor of almost 2), and this should increase the effect due to the difference between the constants G_t(DG) and G_t(MT). The conclusion that G_t^2 is sensitive to the NN potential in the peripheral region ($r > \langle r_t^2\rangle^{1/2} \simeq 1.7$ F, $\langle r_t^2\rangle^{1/2}$ is the rms tritium radius) seems natural to us, since the cluster state should have a peripheral character. Knowing the ratio of V_t and V_s in this region, we can apparently predict which of the cluster states (with a true or with a singlet "deuteron") will have a larger weight in the tritium for any concrete potential.

Potential	$\int(vv)$	$\int(uu)$	$\int(vu)$	P_S, %
MT [1]	0.481	0.115	0.404	2.0
DG [2]	0.012	0.824	0.164	4.7

The possibility of choosing the NN interaction in accordance with G_t^2 is ensured by the fact that G_t^2 can be determined independently and semiphenomenologically from different experiments. These include, e.g., the use of dispersion relations for the forward nd-scattering amplitude [11, 12], a generalized phase-shift analysis of the scattering of a nucleon by a three-nucleon nucleus [13], in which the peripheral phases are determined from the exchange diagrams with the nearest singularities with respect to the momentum transfer, and also a comparative analysis [14] of the direct nuclear reactions (p, d) and (d, t) within the framework of the peripheral model. The indicated semiempirical methods yield for G_t^2 a value close to or somewhat larger than 1 F. The value of G_t^2 obtained in the present paper for the MT potential ($\sim$1.9 F) is close to the semiempirical estimate and practically coincides with the coupling constant for the Reid potential ($G_t^2 \simeq 2$ F).

[1] R.A. Malfliet and I.A. Tjon, Nucl. Phys. A127, 161 (1969).
[2] G. Darewich and A.E.S. Green, Phys. Rev. 164, 1324 (1967).
[3] L.D. Blokhintsev and E.I. Dolinskii, Yad. Fiz. 5, 797 (1967) [Sov. J. Nucl. Phys. 5, 565 (1967)].
[4] Yu.V. Orlov, Vestnik MGU, ser. fiz.-astr. 11, 487 (1970).
[5] Yu.V. Orlov, Izv. AN SSSR, ser. fiz. 34, 2201 (1970).

[6] L.D. Blokhintsev and I.A. Shvarts, Vestnik MGU, ser. fiz.-astr. No. 5, 31 (1972).
[7] A.G. Sitenko, V.F. Kharchenko, and N.M. Petrov, Phys. Lett. 28B, 308 (1968); V.F. Kharchenko and N.M. Petrov, Preprint ITF-69-8, Kiev, 1969.
[8] B. Akhmadkhodzhaev, V.B. Belyaev, and E. Wrzecionko, Yad. Fiz. 11, 1016 (1970) [Sov. J. Nucl. Phys. 11, 565 (1970).
[9] B. Akhmadhodzaev, V.B. Belyaev, I. Wrzecionko, and A.L. Zubarev, Preprint JINR F4-5763, Dubna, 1971.
[10] Y.E. Kim and A. Tubis, Phys. Rev. Lett. 29, 1017 (1972).
[11] M.P. Locher, Nucl. Phys. B23, 116 (1970).
[12] L.S. Kisslinger, Phys. Rev. Lett. 29, 505 (1972).
[13] M. Bolsterly and G. Hale, Phys. Rev. Lett. 28, 1285 (1972).
[14] I. Borbei and E.I. Dolinskii, Yad. Fiz. 10, 299 (1969([Sov. J. Nucl. Phys. 10, 173 (1970)].

E R R A T U M

In the article by Yu. V. Orlov and V. B. Belyaev (Vol. 17, No. 7, p. 276) it is necessary to introduce into the formula for the connection between G_t^2 and C^2 (in the footnote on p. 277) a factor 1/2 (furthermore, λ should be replaced by λ^2). Accordingly the constant G_t for a Reid potential is equal to 1 F, and not 2 F as indicated in the article. On p. 278, line 14 from the top, read DG instead of DT.