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INFLUENCE OF TRANSVERSE MAGNETIC FIELD ON LANDAU DAMPING

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1. The presence of an arbitrarily weak transverse magnetic field in the linear theory leads to the paradox wherein the damping vanishes because of continuous Landau resonance $\delta(\omega - kv)$ gives way to discrete resonances $\delta(\omega - n\omega_H)$, where $\omega_H = eH/mc$ and $n = \pm 1$ and ± 2 [1]. This paradox is resolved, in particular, if the limit is taken correctly with allowance for the fact that as H decreases ω_H becomes ultimately smaller than $1/\tau_0$ ($\tau_0 = \sqrt{m/eEk}$ is the period of the oscillations of the trapped particles and E is the amplitude of the electric field), and the discrete resonances overlap, $\Delta\omega \sim 1/\tau_0 > \omega_H$. The nonlinear effects in the damping then become significant.

As is well known, the Landau damping with decrement γ_L determined by the linear theory takes place at $\omega_H = 0$, for a wave of finite amplitude, only at sufficiently short times $t < \tau_0$, and no noticeable absorption of the wave occurs if $\gamma_L\tau_0 \ll 1$ [2 - 4]. The magnetic field changes the distribution function of the particles in the resonant region within a characteristic time $\tau^{REL} \sim (1/\omega_H)\sqrt{eE/kmv_T^2}$ (v_T is the thermal velocity), so that no "plateau" is established, and one can expect the asymptotic damping decrement to be determined by an interpolation formula of the type [5]

$$\gamma = \gamma_L \left(1 + \frac{\tau^{REL}}{\tau_0}\right)^{-1} . \tag{1}$$

Greatest interest attaches to the case of fields that are so weak that $\varepsilon = \tau_0/\tau^{REL} \ll 1$. Even in such fields, as shown below, there occurs, on top of (1), an additional field damping due to the acceleration of the trapped particles along the wave front. The corresponding decrement increases with time and its maximum value can greatly exceed γ_L.

2. In a magnetic field, the particles trapped by the wave, whose energy is $\mathcal{E} < eE/k$, are turned by the magnetic field upon reflection from the boundaries of the potential well and are accelerated in a perpendicular direction, as shown in Fig. 1. The particle velocity in this direction increases linearly with time:

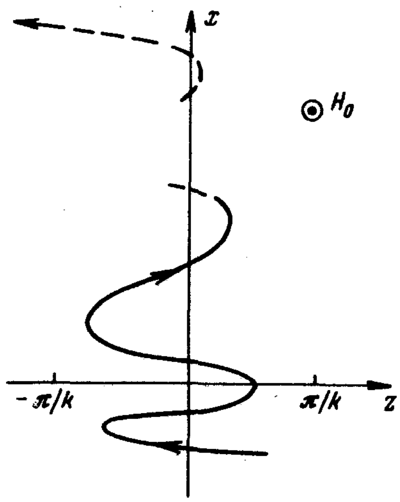

Fig. 1

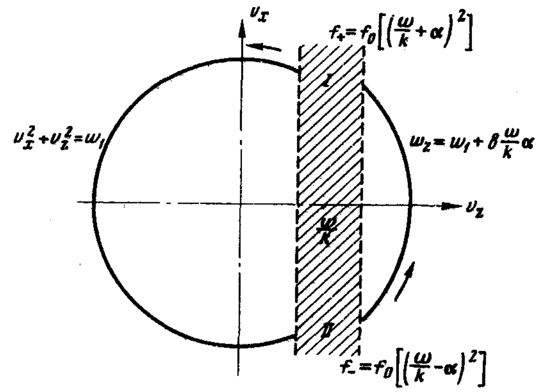

Fig. 2

$$v_x = v_x(0) + \frac{\omega}{k}\omega_H t, \tag{2}$$

until the Lorentz force $ev_x H/c$ exceeds the reflecting force $-eE_z$, as a result of which the particle "tumbles out" of the well.

The particle motion in the direction of the wave propagation is determined from the energy integral

$$\frac{m}{2}\left(\frac{dz'}{dt}\right)^2 - \frac{eE}{k}\cos kz' + m\omega_H \int_0^t v_x \frac{dz'}{dt}\,dt = \mathcal{E}, \tag{3}$$

where we have represented the electric field of the wave in the form $E(t, z) = E \sin kz'$ ($z' = z - \omega t/k$). The magnetic field leads to a shift of the turning points of the captured particle

$$z'_\pm = \pm z_0 + \delta z,\ \cos k(z_0 + \delta z) - \cos k(z_0 - \delta z) = \frac{2m\omega_H}{eE} k z_0 v_x(t) \tag{4}$$

and at $t \sim \tau^{TR}$, when $\cos k(z_0 + z) = -1$ for one of the turning points, the particle leaves the well. From (4) we have the following estimate for τ^{TR}

$$\tau^{TR} \sim \frac{1}{\omega\,\omega_H^2 \tau_0^2} \approx \tau_0 \frac{1}{\epsilon^2 \mu}, \tag{5}$$

where $\mu = (\omega/kv_T^2)\sqrt{eE/km} \ll 1$, corresponding to a small change in the equilibrium distribution function over the velocity interval $\sim 1/k\tau_0$. During that time, the transverse particle energy increases to the large value $\sim mv_T^2/\varepsilon^2$.

The acceleration of the trapped particles in a perpendicular direction is at the expense of the wave energy, and at times $t \lesssim \tau^{TR}$ it leads to a damping of its amplitude, with a decrement that increases with time[1]. The decrement

[1] A similar mechanism of ion acceleration in a transverse magnetic field is possible on the front of a shock wave (see [6]).

for damping by the trapped particles is obtained from the equation

$$\frac{\gamma^{TR}}{4\pi}E^2+\frac{dW^{TR}}{dt}=0, \tag{6}$$

$$W^{TR}=\frac{m}{\lambda}\int dz(0)dv_x(0)\,dv_z(0)f_o[v_x^2(0)+v_z^2(0)]\left(\frac{\omega}{k}\frac{dz'}{dt}+\frac{1}{2}v_x^2\right)$$

is the trapped-particle energy averaged over the wave length, $f_0(v_x^2+v_z^2)$ is their equilibrium distribution function. The change of the longitudinal energy of the trapped particles leads to rapid oscillations of the decrement; these oscillations attenuate at $t \gg \tau_0$. If $t > \tau_0/\varepsilon$, the main factor in W^{TR} is the change of the transverse energy of the particles, and we then obtain from (6) a simple formula for τ^{TR}:

$$\gamma^{TR}=-\frac{8}{\pi}\epsilon^2\mu\sqrt{\frac{E(0)}{E}}\frac{\omega^3}{k}t\int dv_x f_o\left(v_x^2+\frac{\omega^2}{k^2}\right)=-\frac{16}{\pi^2}\gamma_L\sqrt{\frac{E(0)}{E}}\epsilon^2\frac{t}{\tau_o} \tag{7}$$

for a Maxwellian distribution function.

It is important that, unlike the linear Landau damping, which reverses sign when the derivative $\partial f_0/\partial v_z$ ($v_z = \omega/k$) is reversed, the mechanism considered here always leads to wave absorption.

3. The untrapped particles move in a magnetic field along the trajectories shown in Fig. 2. At $\varepsilon \ll 1$, the particle motion in the resonant region $v_z \simeq \omega/k$ is along the trajectory of a pendulum with slowly varying energy

$$F\left(\xi,\frac{1}{\kappa}\right)=F\left(\xi_o,\frac{1}{\kappa}\right)+\frac{K\left(\frac{1}{\kappa}\right)}{\tau_o}\int_{t_o}^{t}\frac{\kappa}{K\left(\frac{1}{\kappa}\right)}dt,$$

$$\xi=\frac{kz'}{2},\quad \kappa^2=\frac{\mathcal{E}+\frac{eE}{k}}{2eE/k},\quad \kappa E\left(\frac{1}{\kappa}\right)-1=-\frac{\pi}{4}\frac{v_{xo}}{\sqrt{eE/km}}\omega_H(t-t_o) \tag{8}$$

$F(\xi,\kappa)$ is an elliptic integral of the first kind, and $K(\kappa)$ and $E(\kappa)$ are complete elliptic integrals. In the formula for $\kappa(t)$ we took into account the fact that at $\mu \ll 1$ the transverse velocity of the particle remains approximately constant, $v_x \simeq v_{x0}$, during the time of flight through the resonant region τ_0/ε. Outside the resonant region, the particles move along constant-energy lines $v_x^2 + v_z^2 = \text{const}$. Making the solutions for these two regions continuous, we obtain relations that connect v_{x0} and t_0 with the initial particle velocities $v_x(0)$ and $v_z(0)$:

$$v_{xo}=v_x(0)\cos\omega_H t_o+v_z(0)\sin\omega_H t_o,$$

$$\frac{\omega}{k}\pm\alpha=v_z(0)\cos\omega_H t_o-v_x(0)\sin\omega_H t_o, \tag{9}$$

$\alpha = (4/\pi)\sqrt{eE/km}$, the $\pm$ sign pertains to the resonant regions I and II, for which $v_{x0} > 0$ and $v_{x0} < 0$, respectively. The change in the energy of the untrapped particle on going through the resonant region is $(m/2)\Delta(v_x^2 + v_z^2) = \pm 4m(\omega/k)\alpha$. Thus, the nonlinear damping of the wave by the untrapped particles is a differential effect which is connected with the difference between the distribution functions in regions I and II:

$$f = f_0\left[v_{x0}^2 + \left(\frac{\omega}{k} \pm \alpha\right)^2\right].$$

The damping decrement γ^{UTR} is determined from the law of energy conservation, and its value at times t satisfying the condition $\tau_p/\varepsilon < t < \pi/\omega_H$ is

$$\gamma^{UTR} = \frac{64}{\pi^3} I \gamma_L \epsilon \exp\left[-\frac{m}{2T}\frac{\omega^2}{k^2}\mathrm{tg}^2\frac{\omega_H t}{2}\right],$$

$$I = \sqrt{\frac{2}{\pi}}\int_0^1 \frac{d\kappa}{\kappa^2} K(\kappa)\left[\frac{\pi^2}{4k^3(\kappa)}\frac{E(\kappa)}{1-\kappa^2} - 1\right] \approx 0.86. \tag{10}$$

The mean value of this damping decrement agrees with formula (1). In the derivation of (10) we took into account the fact that the contribution to γ^{UTR} is made only by particles with $c|v_{x0}| > (\omega/k)\tan(\omega_H t/2)$, for which $f_I \neq f_{II}$. The time of damping by the untrapped particles is $\tau^{UTR} \sim (1/\omega_H)(kv_T/\omega) \simeq (\tau_0/\varepsilon\mu)$. The rotation of these particles in the magnetic field causes the decrement to reverse the sign at $t > \pi/\omega_H$ and to be determined by the formula

$$\gamma^{UTR}(t) = -\gamma^{UTR}\left(\frac{2\pi}{\omega_H} - t\right), \quad \frac{\pi}{\omega_H} < t < \frac{2\pi}{\omega_H}.$$

At large t, the decrement for the untrapped particles oscillates in time with a period $2\pi/\omega_H$. However, after a very long time, $t \gtrsim (1/\omega_H)(\omega/\sqrt{eEk/m}) \gg 1/\omega_H$, the phases of the Larmor rotation of the untrapped particles diverge, owing to the non-synchronism of the motion through the resonant region, and the oscillations indicated above vanish.

Comparing (7) and (10) we find that at $t > \tau_0/\varepsilon$ the main factor is damping by the trapped particles. The change of the field amplitude with time is then determined from the relation

$$E^2(t) \approx E^2(0)\left[1 - \frac{16}{\pi^2}\gamma_L \epsilon^2(0)\frac{t^2}{\tau_0(0)} + \dots\right],$$

which is valid up to $t \lesssim \tau^{TR}$.

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ERRATUM

Article by Yu. A. Golubkov et al., Vol. 17, No. 3, p. 110, the initials of one of the authors were incorrectly given; they should be G. V. Rozhnov.