GRAVITATIONAL INTERACTION OF ZERO-MASS PARTICLES

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It is shown that in a gravitational interaction of zero-mass particles the energy momentum of the system is altered during the collision, but returns to the initial value as a result of the entire collision. It is noted that in principle there can exist macroscopic bodies moving with the speed of light and having on the whole a zero rest mass.

Let us examine within the framework of the classical theory the question of the gravitational field produced by massless particles, and the gravitational interaction of such particles with one another and with particles of finite mass. Even though the gravitational interaction of elementary particles (neutrinos and photons) is negligibly small, this question is of fundamental interest, particularly in connection with those unique singularities which, as will be shown below, this interaction possesses. In addition, we shall show below that in principle there can exist in nature macroscopic objects consisting of particles of finite mass, but behaving as a whole like bodies of zero mass moving always with the speed of light. For such bodies, the gravitational interaction would be the fundamental one.

1. Let the straight line $x = y = 0$, $z = t$ be the trajectory of a massless particle with four-momentum $p^i = (\varepsilon, p)$ with $p_i p^i = 0$. The energy-momentum tensor is then $T_{ik} = (P_i P_k/\varepsilon)\delta(t - z)\delta(\vec{r})$, where $\vec{r} = (x, y)$ is the two-dimensional radius vector. It is required to solve the linearized Einstein equations (see [1], Sec. 105)

$$\Box\psi_{ik} = \frac{16\pi\kappa}{\varepsilon} p_i p_k \delta(t - z)\delta(r), \tag{1}$$

where κ is the Newtonian gravitational constant, $\psi_i^k = h_i^k - (1/2)\delta_i^k h_\ell^\ell$, and h_{ik} are small corrections to the metric tensor. The solution should satisfy the following important condition. Namely, it should be invariant against those Lorentz transformations which leave the particle momentum $p^i = (\varepsilon, 0, 0, \varepsilon)$ invariant. In the opposite case relativistic invariance would in essence be violated, since in the classical theory the particle should be regarded as point-like and its only characteristic is the 4-momentum, and there would exist Lorentz transformations that do not change the momentum but change the field produced by the particle. The symmetry group of the isotropic vector is the group E(2) of motions (translations and rotations) of the plane (see [2]). It is simplest to represent in the coordinates ζ, η, u and v, which are connected with the Cartesian coordinates by the relations

$$x = \eta u, \quad y = \eta v, \quad \eta = t - z, \quad \xi \equiv t + z = \zeta + \eta(u^2 + v^2). \tag{2}$$

The transformations of the group E(2) are those Lorentz transformations which reduce to translations and rotations in the (uv) plane at constant ζ and η. It is seen from (2) that such transformations do not change the vectors with $t - z = \eta = 0$, i.e., they do not change the momentum p^i. From the homogeneity of space-time it follows that the solution can depend only on η and r, but at $\eta \neq 0$ the dependence on r is eliminated by the E(2) symmetry requirement. Since a solution that depends only on η describes a free gravitational wave having no connection with the particle, we should assume that the quantities ψ_{ik} differ from zero only at $\eta = 0$. The corresponding solution of (1) is[1)]

$$\psi_{ik} = h_{ik} = \frac{8\kappa}{\epsilon} p_i p_k \delta(\eta) \ln \frac{r}{r_0} , \tag{3}$$

where r_0 is an arbitrary constant. The field differs from zero only in a plane passing through the particle, perpendicular to the direction of its motion. Formula (3) for an arbitrary straight-line trajectory can be written in the following relativistically-invariant form

$$h_{ik}(x^\ell) = 4\kappa p_i p_k \delta(p_\ell x^\ell - p_\ell x_0^\ell) \ln \frac{(x^m - x_0^m)(x_{0m} - x_m)}{r_0^2}, \tag{4}$$

where we have introduced the quantities x_0^i, which represent some 4-point on the trajectory. They are defined to within the transformation $x_0^i \to x_0^i + \alpha p^i$, where α is an arbitrary constant. In view of $p_i p^i = 0$, the solution (4) is invariant to this transformation. The concrete value of the constant r_0 does not affect the physical consequences. Indeed, if we introduce the variable $\xi' = \xi + 8\kappa\varepsilon\theta(\eta)\ln(r/r_0)$, where $\theta(\eta) = 0$ at $\eta < 0$ and $\theta(\eta) = 1$ at $\eta > 0$, then the interval corresponding to the metric (3) takes the form

$$ds^2 = d\eta\, d\xi' - (8\kappa\epsilon/r)\theta(\eta)\, d\eta\, dr - dr^2 - r^2 d\phi^2,$$

i.e., it does not depend on r_0 at all. Here $\phi = \tan^{-1}(y/x)$.

Let us see how the momentum of a test particle is changed on passing through the plane $\eta = 0$ from the region $\eta < 0$ to the region $\eta > 0$. From the Hamilton-Jacobi equation for the action S of a test particle with mass m (in particular, we can have $m = 0$)

$$\frac{\partial S}{\partial \xi}\frac{\partial S}{\partial \eta} - \left(\frac{\partial S}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \phi}\right)^2 + m^2 = 8\kappa\epsilon\delta(\eta)\ln\frac{r}{r_0}\left(\frac{\partial S}{\partial \xi}\right)^2$$

we see that the derivatives of S with respect to ξ and ϕ remain unchanged, i.e., the difference $\omega - k_z$ and k_ϕ remain unchanged, where ω is the energy and $\vec{k}$ is the momentum. The action S experiences a jump equal to $\Delta S = -4\kappa\varepsilon(\omega - k_z)\times \ln(r/r_0)$ in the plane $\eta = 0$. Differentiating the last formula with respect to r, we obtain the increment of the r-component of the momentum

$$\Delta k_r = -(4\kappa\epsilon/r)(\omega - k_z). \tag{5}$$

The change of the quantity $\omega + k_z$ is determined from the condition $k_i k^i \equiv m^2$.

The process of collision of two massless particles in the c.m.s., in which the particles have equal energies ε and move opposite to each other with a certain impact distance r, can be described as follows. The particles move freely until they are in the same plane perpendicular to the direction of motion. At this instant they change their momenta jumpwise, after which they again move

[1)] Tolman, Ehrenfest, and Podolsky [3] (see also [4]) have considered the problem of the gravitational field of a packet of electromagnetic waves, equivalent to the problem of a point-like massless particle. They, however, have used a solution in the form of retarded potentials, which is unsatisfactory because of the absence of E(2) invariance.

freely. From (5) we obtain (in this case $\omega = \varepsilon$ and $k_z = -\varepsilon$) the scattering angle $\chi = -\Delta k_r/\varepsilon = 8\kappa\varepsilon/r$ and the differential cross section $d\sigma = 2\pi r dr = 2\pi(8\kappa\varepsilon)^2 d\chi/\chi^3$. Its dependence on the angle is the same as for ordinary particles interacting in accordance with Newton's law.

A unique behavior is possessed by the collision of the massless particles with a resting particle of finite mass. The former moves in a known manner in the Newtonian field of the massive particle, and its momentum changes continuously. The massive particle, on the other hand, acquires jumpwise a momentum $k = 4\kappa\varepsilon m/r$ (ε is the energy of the massless particle, m is the rest mass of the massive particle, and r is the impact distance) at the instant of crossing of the field of the massless particle, after which it moves freely. This momentum coincides exactly with the change of the momentum of the massless particle over the entire period of the collision. The latter is equal to $\varepsilon\Delta\phi$, where $\Delta\phi$ is the known (see [1], Sec. 98) value of the deflection angle of the photon and the gravitational field of a body at rest. Since we should assume in the approximation linear in the field that the energy and momentum of the system are equal to the sum of the energies and momenta of the particles, we encounter here nonconservation of the energy momentum during the course of the collision. The same takes place also when massless particles collide with one another in a reference frame different from the c.m.s., when the particles have different momenta at the different instants of time. It is important, however, that as the result of the entire collision the energy and the momentum of the system remain unchanged.

2. The gravitational field in the region behind the zero-mass body is equal to zero only in the linear approximation. Let us calculate this field on the basis of Einstein's exact nonlinear equations. The most general metric, which is invariant against the group E(2), can be written in the form

$$ds^2 = A(\eta, \zeta)\, d\eta\, d\zeta - B(\eta, \zeta)(du^2 + dv^2) \qquad (6)$$

The Galilean metric corresponds in the coordinates (2) to $A = 1$ and $B = \eta^2$. Substituting (6) into the field equations in vacuum, we obtain $B = (2/A)^2 = \alpha\eta + \beta\zeta + \gamma$, where α, β, γ are arbitrary constants. After the transformation $\alpha\eta + \beta\zeta + \gamma \to \eta$, we obtain ultimately

$$ds^2 = d\zeta d\eta + (\eta_0/\eta)\, d\zeta^2 - \eta^2(du^2 + dv^2), \qquad (7)$$

where $\eta_0 > 0$ is a constant. The metric (7) reduces to the Kasner metric with indices (p_1, p_2, p_3) (see [1], Sec. 103) equal to $(-1/3, 2/3, 2/3)$ and has a physical singularity at $\eta = 0$. As $\eta \to \infty$, the metric tends to the Galilean form, and therefore the coordinate η increases with increasing distance from the body. The boundaries of the light cone for motions with $du = dv = 0$, i.e., particularly for motions along the straight line along which the body moves, correspond, as seen from (7), to the equations $d\zeta = 0$ and $d\eta = -(\eta_0/\eta)d\zeta$. It is clear therefore that at finite η inside the future cone there are directions corresponding to a decrease of η. Thus, test particles of finite mass, located at a finite distance from the body, can overtake it in spite of the fact that the body moves with the speed of light relative to the remote observer. All the more, they can be addressed relative to the observer. It follows therefore that a zero-mass body can consist of particles with finite mass.

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SURFACE EXCITONS OF ELECTRON-HOLE TYPE AND COLLECTIVE PHENOMENA ASSOCIATED WITH THEM

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We discuss the question of surface excitons of the electron-hole type (their binding energy is calculated in the "macroscopic" approximation). Mention is made of a number of possibilities that arise when the concentration of the surface excitons increases.

The problem of surface levels as applied to metals and semiconductors has been under discussion since 1932 [1], but there are still many obscure aspects from the theoretical and particularly from the experimental point of view [2]. At the same time, an increased interest in surface levels is already observed and one can expect even more in the nearest future, owing to the development of adequate experimental methods. We wish to call attention here to the possible existence of surface excitons of the electron-hole type (i.e., excitons of the Wannier-Mott type (this circumstance was already pointed out in [3, 4], but the question was not discussed in detail). In the simplest case of isotropic parabolic bands, the binding energy of a surface exciton is determined from the two-dimensional Schrodinger equation

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+V(x,y)\right]\psi(x,y)=E\psi(x,y). \quad (1)$$

Here $m^{-1} = m_e^{-1} + m_h^{-1}$, where m_e and m_h are the effective masses of the electrons and holes in the surface bands. This equation is exact, as is the equation for three-dimensional Wannier-Mott excitons (see, e.g., [5]). The spatial inhomogeneity of the problem is taken into account when determining the type of the operator of the electron-hole interaction V(x, y). Let the region near the boundary in which the surface states are localized have a thickness of the order of b. When b >> a (a is of the order of the lattice constant), V(x, y) can be determined by using a macroscopic image method, for in this case the image is located far enough from the separation boundary. Let the dielectric constant of the medium be ε_1 at $z < 0$ and ε_2 at $z > 0$. The energy $U(\rho, z_e, z_h)$ of the electrostatic interaction of an electron and hole located at points with coordinates x_e, y_e, and z_e and x_h, y_h, and z_h is equal to $-(e^2/\varepsilon_2)[1/R + \alpha/R']$ if $z_e > 0$ and $z_h > 0$ to $-(e^2/\varepsilon_1)[1/R - \alpha/R']$ if $z_e < 0$ and $z_h < 0$, and to $-2e^2/(\varepsilon_1 + \varepsilon_2)$ if $z_e z_h < 0$. Here $\alpha = (\varepsilon_2 - \varepsilon_1)/(\varepsilon_2 + \varepsilon_1)$, $\rho = [(x_e - x_h)^2 + (y_e - y_h)^2]^{1/2}$, $R = [\rho^2 + (z_e - z_h)^2]^{1/2}$, and $R' = [\rho^2 + (z_e + z_h)^2]^{1/2}$. At sufficiently large distance between the electron and the hole, the operator V(x, y) is given by the diagonal matrix element (see [5])

$$\int dx_e\, dy_e\, dz_e\, dx_h\, dy_h\, dz_h\, |a_{oo}(x_h, y_h, z_h)|^2 |b_{xy}(x_e, y_e, z_e)|^2 U(\rho, z_e, z_h), \quad (2)$$

where a_{00} and b_{xy} are the wave function of the surface bands in the Wannier