For $P_\perp \gg \mu_\pi$, however, the situation is at present uncertain, in connection with the experimental data recently obtained at CERN (ISR) in the measurement of the cross section of the reaction $pp \to \pi^0 + X$ at large $P_\perp$ [10]. It may turn out that the dependence of the cross sections of the inclusive hadronic processes on P will be not exponential but power-law [11], and that in the range of values $E \gg K_\perp \gg \mu_\pi$ the two-photon mechanism of hadron production may in principle become a rather substantial source of photons emitted at large angles in e^+e^- collisions.

A detailed exposition of the group of questions considered here will be published.

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SUM RULES IN ELECTRON SCATTERING BY NUCLEI

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New sum rules are formulated for the scattering of electrons by nuclei. Some of these sum rules are independent of the model in the single-particle approximation for the nuclear current. Such sum rules are attractive for an experimental determination of the contributions of the non-single-particle currents.

1. Let $q_\mu = (\vec{q}, i\omega)$ be the 4-momentum transferred by the electron to the nucleus, and let θ and $\varepsilon_{i(f)}$ be the electron scattering angle and its initial

(final) energy. Let furthermore A(q, ω) and B(q, ω) be the form factors that determine the differential cross section for the scattering of an unpolarized electron by an unpolarized nucleus at rest in the one-photon exchange approximation neglecting the electron mass

$$\sigma^{-1}_{\text{Mott}}(\theta)\, d^2\sigma/d\Omega\, d\epsilon_f = A(q, \omega) + B(q, \omega)\, \mathrm{tg}^2\theta/2 . \tag{1}$$

We consider the quantities[1)]

$$\sigma_\ell(q^2) = \int [A(q, \omega) - {}^1/_2(1-\omega^2/q^2)B(q, \omega)] \times \tag{2}$$

$$\times (1-\omega^2/q^2)^{-2}(G_{Ep}(q^2)/G_{Ep}(q^2-\omega^2))^2 d\omega ,$$

$$\sigma_{\ell_1}(q^2) = \int \omega [A(q, \omega) - {}^1/_2(1-\omega^2/q^2)B(q, \omega)] \times \tag{3}$$

$$\times (1-\omega^2/q^2)^{-2}(G_{Ep}(q^2)/G_{Ep}(q^2-\omega^2))^2 d\omega ,$$

$$\sigma_{\ell_2}(q^2) = \int \omega^2 [A(q, \omega) - {}^1/_2(1-\omega^2/q^2)B(q, \omega)] \times$$

$$\times (1-\omega^2/q^2)^{-2}(G_{Ep}(q^2)/G_{Ep}(q^2-\omega^2))^2 d\omega , \tag{4}$$

$$\sigma_t(q^2) = \int B(q, \omega)\, (G_{Ep}(q^2)/G_{Ep}(q^2-\omega^2))^2 d\omega . \tag{5}$$

The quantities (2) - (5) are suitable for the formulation of the sum rules because they can be calculated both from the experimental data and theoretically. The experimental procedure should consist of obtaining the cross sections (1) at fixed q and θ as functions of ω (see [1 - 4]) and subsequently separating the form factors A(q, ω) and B(q, ω) for each value of ω. Theoretical summation in (2) - (5) yields

$$\sigma_\ell(q^2) = (2J_i+1)^{-1} \sum_{M_i} \langle \Psi_i^{J_i M_i} | Q^+ Q | \Psi_i^{J_i M_i} \rangle |_{\omega=0} , \tag{6}$$

$$\sigma_{\ell_1}(q^2) = (2J_i+1)^{-1} \sum_{M_i} \langle \Psi_i^{J_i M_i} | \frac{q}{2}(Q^+ J_\ell + J_\ell^+ Q) | \Psi_i^{J_i M_i} \rangle |_{\omega=0} , \tag{7}$$

$$\sigma_{\ell_2}(q^2) = (2J_i+1)^{-1} \sum_{M_i} \langle \Psi_i^{J_i M_i} | q^2 J_\ell^+ J_\ell | \Psi_i^{J_i M_i} \rangle |_{\omega=0} , \tag{8}$$

$$\sigma_t(q^2) = (2J_i+1)^{-1} \sum_{M_i} \langle \Psi_i^{J_i M_i} | J_t^+ J_t | \Psi_i^{J_i M_i} \rangle |_{\omega=0}$$

$$J_\ell = q^{-1}(\mathbf{q}\mathbf{J}), \quad \mathbf{J}_t = \mathbf{J} - J_\ell q^{-1} \mathbf{q} . \tag{9}$$

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1) In (2) - (5), ∫...dω includes the sum over the discrete levels. For transitions to discrete levels, $d\epsilon_f$ should be omitted from (1), and σ_{Mott} should contain the recoil factor. The nuclear form factors G_{Ep}, G_{En}, G_{Mp}, and G_{Mn} are used in (2) - (5) and subsequently.

$\Psi_i^{J_i M_i}$ in (6) - (9) is the Ψ-function of the initial state of the nucleus in its c.m.s., and Q and $\vec{J}$ are the operator of the interaction of the field produced by the electron with the charge and current of the nucleus, and correspond to the electron-nuclear interaction (cf. the notation in [2])

$$H' = q_\mu^{-2}\, 4\pi \ell^2 < u(\epsilon_f) \mid Q(q,\omega) - \vec{\alpha}\, J(q,\omega) \mid u(\epsilon_i) > .$$

The sum rules (6) - (9) have definite advantages over the known sum rules [1, 2] in that they make it possible to investigate separately the contributions of the nuclear charge, of the longitudinal current, and of the transverse current. Furthermore, they do not contain the values of $\bar{\omega}$ and $\overline{\omega^2}$ averaged over the spectrum, which are difficult to calculate accurately[2]), and do not make use of the approximation for the nucleon form factors

$$F_{1p}(q_\mu^2) = F_{2p}(q_\mu^2) = F_{2n}(q_\mu^2) = f(q_\mu^2); \quad F_{1n}(q_\mu^2) = 0, \tag{10}$$

which is now known to contain errors $\sim(q/M)^2$ (M is the nucleon mass). The separation of the form factors of [5, 4] in the sum rule of [1, 2] transforms the latter into two sum rules, the simpler of which [5] is analogous to the sum rule (9), but makes use of the approximation (10). We propose, as described above, to interchange the order of the operations of summing over dω and separating the form factors; this makes it possible to find the sums (2) - (4).

2. Assume for the nuclear current, as usual, the single-particle approximation

$$Q = \sum_{i=1}^{A} \hat{\rho}_i, \quad J = \sum_{i=1}^{A} \hat{j}_i . \tag{11}$$

As is well known, accurate to M^{-2} inclusive, we have

$$\hat{\rho}_i = \left[\left(1 - \frac{q^2}{8M^2}\right)\hat{a}_i^E - i\,\frac{2\hat{a}_i^M - \hat{a}_i^E}{4M^2}\,\vec{\sigma}_i\,[\,q \times p_i\,]\right]\exp(i q r_i), \tag{12}$$

$$\hat{j}_i = (2M)^{-1}[\,\hat{a}_i^E(p_i e^{iqr_i} + e^{iqr_i} p_i) + \hat{a}_i^M[\vec{\sigma}_i \times q]\, e^{iqr_i}\,], \tag{13}$$

where

$$\hat{a}_i^{E(M)}(q_\mu^2) = G_{\substack{Ep\\(Mp)}}(q_\mu^2)\,\frac{1 + \tau_z(i)}{2} + G_{\substack{En\\(Mn)}}(q_\mu^2)\,\frac{1 - \tau_z(i)}{2}. \tag{14}$$

We consider the sum rule for $\sigma_{\ell 1}$ (7). It can be shown that in the approximation (11) the following relation holds true:

$$\sigma_{\ell 1} = (2J_i + 1)^{-1} \sum_{M_i} < \Psi_i^{J_i M_i} \mid \frac{1}{2} \sum_{i=1}^{A} [\,\hat{\rho}_i^{\dagger}(q\,\hat{j}_i) + (q\,\hat{j}_i^{\dagger})\hat{\rho}_i\,] \mid \Psi_i^{J_i M_i} > \Big|_{\omega = 0} \tag{15}$$

On the other hand, the contribution made to $\sigma_{\ell 1}$ by terms corresponding to the products of charges and currents of different particles turns out to be equal

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[2]) The estimate of $\bar{\omega}$ given in [2] is valid only at sufficiently large q, while the estimate of $\overline{\omega^2}$ is incorrect.

to zero because of requirements connected with the time-reversal operation. Substituting (12) and (13) into (15) we obtain the simple result

$$\sigma_{\ell_1}(q^2) = (q^2/2M)\, Z(G_{E_p}(q^2))^2. \tag{16}$$

Thus, in the single-particle approximation the sum (3) is expressed only in terms of the form factors of the free nucleons. The non-single-particle currents make an additional model-dependent contribution to σ_{ℓ_1}. The question of the character of this contribution will be investigated in a separate paper[3)]. We note here only that this contribution can reach values that are appreciable in comparison with the contribution (16). Formula (16) is similar to the formula for the differential slowing-down ability in the theory of ionization losses of electrons on atoms.

3. It is possible to make the results (6), (8), and (9) independent of the form of $\Psi_i^{J_iM_i}$ for the sums (2), (4), and (5) by assuming (11) - (14) in the case of the lightest nuclei. It is thus easy to verify that, with a high degree of accuracy,

$$\sigma_\ell^{^2H}(q^2) = \left(1 - \frac{q^2}{4M^2}\right)(G_{E_p}(q^2))^2 + \left(1 + \frac{q^2}{4M^2}\right)\frac{2G_{E_n}(q^2)G_{E_p}(q^2)G_0^{^2H}(4q^2)}{G_{E_p}(4q^2) + G_{E_n}(4q^2)} \tag{17}$$

where $G_0^{^2H}$ is the monopole charge form factor of elastic e-d scattering. The quality $G_0^{^2H}$ is determined directly by measuring the polarization of elastically-scattered deuterons [6]. The model-independent relation (17) may be useful in the question of the electric form factor G_{En} of the neutron.

Model-independent relations such as (17) can be obtained also for A = 3 and 4, by using the transformation of Fabre de la Ripelle [7].

We obtain, for example

$$\sigma_\ell^{^4He}(q^2) \approx 2\left(1 - \frac{q^2}{4M^2}\right)(G_{E_p}(q^2))^2 + \left(1 + \frac{q^2}{8M^2}\right)F^{^4He}\left(\frac{8}{3}q^2\right) \times$$
$$\times \frac{(G_{E_p}(q^2))^2 + 4G_{E_n}(q^2)G_{E_p}(q^2)}{G_{E_p}\left(\frac{8}{3}q^2\right) + G_{E_n}\left(\frac{8}{3}q^2\right)} \quad . \tag{18}$$

Here $F^{^4He}$ is the elastic form factor of e-α scattering.

4. The Coulomb energy of the nucleus with allowance for the finite dimensions of the nucleons is

$$E_{Coul} = \frac{A(A-1)}{2}\frac{e^2}{2\pi} < \Psi_i | \int d^3q\, q^{-2}\, \hat{a}_1^E\, \hat{a}_2^E \exp i\, \mathbf{q}\, \mathbf{r}_{12} | \Psi_i > . \tag{19}$$

On the other hand, let us substitute (11) and (12) in (6) and sum over M_i. Neglecting the contribution of the spin-orbit interaction and the quantity G_{En}^2, we obtain

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3) The study of this question was suggested to the author by I.S. Shapiro.

$$\sigma_{\ell}(q^2) = \left(1 - \frac{q^2}{4M^2}\right)\left[Z(G_{E_p}(q^2))^2 + A(A-1) < \Psi_i \,|\, \hat{a}_1^E \hat{a}_2^E \, i_0 \, (qr_{12})\Psi_i >_{|\omega = 0}\right]. \qquad (20)$$

Comparing (19) and (20) we obtain

$$E_{\text{Coul}} = (e^2/\pi) \int_0^{\infty} dq[\sigma_{\ell}(q^2)(1 + q^2/4M^2) - Z(G_{E_p}(q^2))^2]. \qquad (21)$$

With the aid of (21) we can determine, in principle, the Coulomb energies of the nuclei from experiments on inelastic scattering of electrons.

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CHARGE ASYMMETRY IN DECAYS OF THE SYSTEM $K^0\overline{K}^0$ PRODUCED VIA A ϕ^0 RESONANCE

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The system $K^0\overline{K}^0$, which is produced in the annihilation reaction (e^+e^-, $p\overline{p}$), has highly unique properties in that subsequent decays of the two neutral kaons are not independent; this is particularly valuable for the study of effects of CP violation (these questions were discussed in [1 - 5], and the program for the corresponding investigations with the VEPP-2M apparatus of our Institute was reported in [6]). The correlation of these two decays is a pure effect of quantum-mechanical coherence, first noted in the famour paper of Einstein, Podolsky, and Rosen [7].

The system $K^0\overline{K}^0$ is produced, with high probability, in the reaction

$$e^+e^- \to \phi^0 \to K^0\overline{K}^0, \qquad (1)$$

in which an antisymmetrical state is produced (angular momentum I = 1, parity P = -1)

$$\psi_a = \frac{1}{\sqrt{2}}[\,|K^0(1)\overline{K}^0(2)> - |K^0(2)\overline{K}^0(1)>] = \frac{1}{\sqrt{2}}[\,|L(1)S(2)> - |L(2)S(1)>], \qquad (2)$$

where $|L>$ ($|S>$) $\equiv |K^0_{L(S)}>$. The cross section of the process (1) at resonance ($2\varepsilon = m_\phi$) is $\sigma_R = 1.3 \times 10^{-30}$ cm^2, so that at a luminosity $L = 10^{31}$ cm^{-2}sec^{-1} (which is apparently quite realistic for second-generation installations) approximately 13 $K^0\overline{K}^0$ pairs will be produced per second. The angular distribution in the reaction (1) is $\sigma(\theta) \propto \sin^2\theta$ (θ is the angle between the vectors $\vec{p}_K$ and