$$a \approx \eta_g^{-1/4}(\eta/\eta_o - 1)(2\xi/\eta_o + \eta/\eta_o - 1)^{-1} \ldots \quad (12)$$

In the case when $\eta/\eta_0 \gg 1$ and $\xi/\eta_g \gg 1$ we have

$$a \approx \eta_g^{-1/4}\eta(2\xi)^{-1} \ldots\ldots \quad (13)$$

or, using (4) and (10), we obtain

$$j/j_o \approx 4\sqrt{\eta_o/\eta_g}\,\eta^2(3\xi)^{-2}. \quad (14)$$

Equation (14) yields the sought approximate dependence of the beam potential η on the modulation depth ξ and on the ion current density $j(\xi, \eta)$.

For example, for $\eta = \eta_g = 10\eta_0$, $\eta_0 = 3 \times 10^{-4}$, $\xi = 6 \times 10^{-2}$, $u = 3 \times 10^4$ V, $T_e = 2$ V, $A = 20$ cm, $\lambda = 10^3$ cm, $M_2 = 28$, and $M = 56$ we obtain $j_0 = 10$ A/cm^2, $j = 3.5$ mA/cm^2, and the total ion current per centimeter of width is $I = jA = 70$ mA/cm. These figures are in satisfactory agreement with the characteristics of ion beams in electromagnetic isotope separators [2 - 4].

It is useful to note that under the natural limitation $\eta \leq \eta_g$ = const we obtain from (14) the condition

$$jA\lambda\xi^2 u^{-5/2} = \text{const}. \quad (15)$$

Consequently, at a given potential η_g, the total current does not depend on the height of the beam, and the permissible current density increases with decreasing height. This may be the reason why it was impossible to increase the productivity of electromagnetic separation installation by increasing the beam height.

In conclusion, I am grateful to Yu.S. Popov and V.S. Erofeev for useful discussions.

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EFFECT OF SCREENING ON THE CRITICAL CHARGE OF A NUCLEUS

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The critical charge Z_c was calculated for a "bare" nucleus, i.e., without allowance for the screening of the Coulomb field $V(r) = -Z\alpha/r$ by the electron shell. The screening weakens the attraction of the electron to the nucleus:

$$V(r) = -\frac{Z\alpha}{r}\chi(r), \quad 0 < \chi(r) < 1, \quad (1)$$

and Z_c increases correspondingly. Here $\chi(0) = 1$ and (for a neutral atom)

$\chi(\infty) = 0$. An estimate of this effect is particularly important in connection with experiments on the spontaneous production of positrons in heavy-nucleus collisions (of the U + U type). In fact, the total charge of the nuclei $Z_1 + Z_2$ is only 15 - 20 units larger than the critical charge $Z_c \simeq 170$ calculated without allowance for screening. An increase of Z_c by 10 - 20 units would therefore make such an experiment (using the presently-known heavy elements) very difficult or generally impossible.

Of course, there is no need to take screening into account if one works with bare nuclei. However, the experimental difficulties with colliding beams of fully stripped nuclei are too large. As shown in [4], spontaneous positron production occurs when only one of the colliding nuclei (Z_1) is bare, and the other (Z_2) has a normal electron shell, if $Z_1 \geq Z_2$. The cross section for e^+ production has in this case practically the same value as for the bare nuclei Z_1 and Z_2. This remark uncovers the possibility of performing experiments with an ordinary heavy target, thereby greatly facilitating the experiment. In this case, however, the quasimolecule produced when the nuclei come close together is certainly surrounded by an electron shell, so that it is necessary to estimate the change of Z_c due to screening.

In view of the difficulty of calculating Z_c in the two-center problem we consider, in order to determine the scale of the screening correction, a model problem with a spherical superheavy nucleus. We choose the function χ in (1) in accord with the Thomas-Fermi equation

$$\chi = \chi(\beta r), \qquad \beta = \frac{Z^{1/3}}{137\, b} = 0.0425\,(Z\alpha)^{1/3} \tag{2}$$

($\hbar = c = m_e = 1$; the remaining notation is the same as in [5]). Although at $Z > 137$ we have $v \simeq c$ for the inner electrons, most electrons are located at distances $r \gtrsim 137 Z^{-1/3} \gg 1$ from the nucleus, thus justifying the use of the nonrelativistic Thomas-Fermi model. Dirac's equation with potential (1) was solved numerically by a phase method that reduces briefly to the following. We make the following substitution in the Dirac system for the radial functions g and f at $\varepsilon = \kappa = -1$ (corresponding to the $1s_{1/2}$ level at the edge of the lower continuum):

$$g = a\cos\theta, \qquad f = \frac{2}{3}\, a\left(r\cos\theta - \frac{1}{r^2}\sin\theta\right), \tag{3}$$

where $a(r)$ and $\theta(r)$ are new unknown functions (amplitude and phase). We obtain for the phase the nonlinear equation

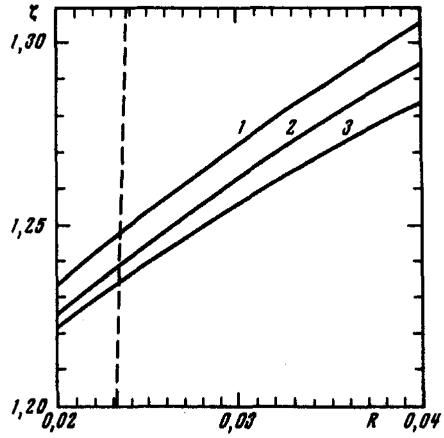
$$\frac{d\theta}{dr} = -V(r)\left[\frac{3}{2} r^2\cos^2\theta + \frac{2}{3}\left(r^2\cos\theta - \frac{\sin\theta}{r}\right)^2\right] \tag{4}$$

with initial condition $\theta(0) = 0$. For an attractive potential, θ is a monotonically increasing function of r. Denoting the asymptotic value of the phase by η:

$$\eta = \lim_{r\to\infty}\theta(r), \tag{5}$$

we obtain a condition[1)] that determines Z_c for the levels $ns_{1/2}$:

[1)] The derivation of this condition, and also a generalization of the phase equation (4) to include states with arbitrary κ, will be published separately.



$$\eta = \left(n + \frac{1}{2}\right)\pi . \qquad (6)$$

Equations (4) and (6) are well suited for computer calculations. A few words concerning the screening function $\chi(x)$, where $x = \beta r$. The use of the tables [5, 6] of $\chi(x)$ turned out not to be very convenient, since these tables are not detailed enough. For $x < 0.4$ we expanded χ in powers of $\sqrt{x}$:

$$\chi(x) = 1 + \sum_{\nu \geqslant 1} a_\nu x^\nu , \qquad (7)$$

where $a_1 = -1.588$, $a_{3/2} = 4/3$, $a_2 = 0$, $a_{5/2} = 2a_1/5$, $a_3 = 1/3$, $a_{7/2} = 3a_1^2/70$, and $a_4 = 2a_1/15$ (terms with $\nu > 4$ were discarded). For $x > 0.4$, we used the Sommerfeld approximation $\chi(x) \simeq [1 + (\gamma x)^\lambda]^{-\mu}$ with suitably chosen parameters γ, λ, and μ (see [6]). The deviation of these approximations from the values obtained for $\chi(x)$ by exact numerical calculations is less than 1% for all x, and in the region $r < 5$, which is the most important for the determination of Z_c from the phase equation (4), this deviation is <0.1%.

The results of the calculation of Z_c are shown in the figure, where $\zeta = Z_c/137$ and R is the nuclear radius in units of $\hbar/m_e c = 386$ F. The potential inside the nucleus was chosen in the form $V(r) = -Z\alpha[3 - (r/R)^2]/2R$, corresponding to a uniform charge density. Curves 2 and 3 correspond to an unscreened nucleon. Curve 3 is the result of a calculation [2] in which the logarithmic derivative rG'/G was determined by using the approximation $R \ll 1$, while curve 2 was calculated from the exact equation (4). Finally, curve 1 takes the screening (1) into account. By specifying a definite dependence of R on Z, one can easily determine the critical charge of the nucleus. If we put $R = r_0 A^{1/3}$, where $r_0 = 1.2$ F and $A = 2.5$ Z (see the dashed curve in the figure), then $Z = 1.247$, 1.238, and 1.234 for curves 1, 2, and 3, respectively.

The net result is $Z_c = 171$; allowance for $\chi(x)$ increases the critical charge by only $\Delta Z_c = 1.2$ (the transition from curve 2 to curve 1). When the bare nucleus collides with a neutral atom, the effect of the screening on Z_c is even smaller, since the electron shell of the joined atom is only half-filled.

It follows hence, first, that the values of Z_c from the two-center problem can be used in the calculation of positron production in collisions of heavy nuclei[2)]; second, the approximation [2] $R \ll 1$ in the internal region $r < R$ results in good accuracy. We note, incidentally, that the solution of the Dirac equation on the basis of the phase equation (4) makes this approximation less essential than before.

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CRITICAL BEHAVIOR OF NONPLANAR ISING MODEL IN STRONG ANTIFERROMAGNETIC INTERACTION ALONG THE DIAGONALS

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It is shown that at small $\lambda = J_1/|J_2|$, where J_1 and J_2 are the constants of the interactions along the sides and along the diagonals, the shift of the transition temperature and the change of the critical exponents are proportional to λ^2. The exponents satisfy in this case the scaling hypothesis.

Only a few models in which a phase transition takes place have been solved exactly to date. These are, first, different variants of the planar Ising model with nearest-neighbor interaction [1], the Slater model and its modification (the so-called six-vertex or F model) [2], and finally the eight-vertex model without an external field or the Baxter model [3], of which the first two models are particular cases. The next in complexity is the nonplanar Ising model, i.e., the Ising model with intersection of the interactions. It is equivalent to the eight-vertex model in an external field; its solution has not been obtained so far, but it is possible to determine the critical exponents of the nonplanar model in one particular case.

If the interaction along the diagonals of the unit cells are all the same and equal to J_2, while the interactions along the sides are different and equal to J_1 and J_1', then the partition function of the nonplanar model in the absence of an external field depends only on the moduli $|J_1 + J_1'|$ and $|J_1 - J_1'|$ and satisfies the relation

$$Z(|J_1+J_1'|,\ |J_1-J_1'|,\ J_2) = Z(|J_1-J_1'|,\ |J_1+J_1'|,\ -J_2). \tag{1}$$

These symmetry properties can be verified by interchanging rows and columns and changing the directions of the spins in checkerboard order in each second row or column.

The phase diagram of the isotropic nonplanar model ($J_1 = J_1'$), as a function of the temperature and on the relations between the interaction constants, is shown schematically in Fig. 1. In region I there is ferromagnetic ordering (along the diagonals), in region II antiferromagnetic ordering, and region III corresponds to the paramagnetic phase. Point B is determined by the condition that the energies of the ferromagnetic and antiferromagnetic phases be equal at zero temperature ($J_2 = -|J_1|/2$), and at points A and C, i.e., when $|J_1| = 0$, the model breaks up into two identical non-interacting planar Ising sublattices. We are interested in the critical behavior near the point A, when $|J_1| \ll -J_2$, so that the interaction J_1 between the sublattices can be regarded as a small parameter.

It will be more convenient for us, however, using the symmetry relation (1), to consider an equivalent model with ferromagnetic interaction along the diagonals, and with equal but opposite interactions along the sides. This model is shown in Fig. 2: The dark and light points belong to the first and second