

in p-polarized light. This band is missing from the ordinary reflection spectrum at E C. For the extraordinary ray at E C, weak bands were observed in [8] in the reflection spectra at the frequencies 885 cm^{-1} for $\alpha\text{-SiC}(6\text{H})$ and 858 cm^{-1} for $\alpha\text{-SiC}(15\text{R})$, and were attributed by the authors to the presence of an active axial optical phonon.

In our case, the $840 - 950\text{ cm}^{-1}$ band is observed in the cubic $\beta\text{-SiC}$ (see the figure), in which there are no volume modes connected with the anisotropy of the crystal. The band lies between the transverse and longitudinal optical phonons, where the dielectric constant $\epsilon(\nu)$ is negative, this being a necessary condition for the existence of surface phonons. In this region, the IR light penetrates less than $1 - 2$ microns into the crystal.

The surface nature of the band is favored also by the dependence of its position and of its half-width on the gap d_3 between the sample and the MIR element. The maximum absorption is observed at $d_3 \sim 0.6 - 0.8\text{ }\mu$, which is due in our case to the non-ideal optical contact. Improvement of the contact decreases this peak. When the $\beta\text{-SiC}$ sample is located $2 - 4\text{ }\mu$ away from the prism, an absorption peak is observed at a frequency $\nu_s = 936\text{ cm}^{-1}$, which does not change in the indicated range) and with a half-width $6 - 8\text{ cm}^{-1}$ (curve 4 in the figure).

The appreciable width of the $840 - 950\text{ cm}^{-1}$ band is due apparently to radiative broadening in the presence of a small gap. The value of the limiting frequency of the nonradiative phonons at $\nu_s = 951\text{ cm}^{-1}$ was obtained with the aid of the dispersion relations for the surface optical oscillations [1 - 5] and the expression for the dielectric constant without allowance for the damping. When the IR light passes through the MIR element at an angle $\phi_s = 45^\circ$, the theoretical value of ν_s is 945 cm^{-1} (the refractive index of the prism is $n = 3.3$), in good agreement with the experimental $\nu_s = 936\text{ cm}^{-1}$. This verifies the surface nature of the observed band.

We used in the calculations the data of [9] for the static and high-frequency dielectric constants, $\epsilon_0 = 9.72$ and $\epsilon_\infty = 6.52$, with $\nu_{\text{TO}} = 796\text{ cm}^{-1}$.

Certain differences between the calculated and experimental values of ν_s for $\phi_s = 45^\circ$ may be due to failure to take the damping into account in the $\epsilon(\nu)$ dependence.

In conclusion, the authors thank T. I. Kucher and M. P. Lisitsa for taking part in a discussion of the results.

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SELF CONSISTENT FIELD METHOD IN THE THEORY OF PARAMETRIC MAGNON EXCITATION

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Submitted 2 April 1973

ZhETF Pis. Red. 17, No. 10, 590 - 593 (20 May 1973)

The Hamiltonian of the spin system of a ferromagnet placed in a high-frequency magnetic field parallel to a chosen axis has the following form in the spin-wave approximation:

$$\mathcal{H} = \sum_k \epsilon_k a_k^+ a_k + \frac{1}{2} \sum_k (V_k a_k a_{-k} e^{i\omega t} + V_k^* a_k^+ a_{-k}^+ e^{-i\omega t}) + \mathcal{H}_{int} \quad (1)$$

$$\epsilon_k = \sqrt{A_k^2 - |B_k|^2}, \quad V_k = \frac{\mu h}{2\epsilon_k} B_k,$$

where h_0 is the amplitude of the high-frequency field, a_k and a_k^+ are magnon Bose operators, and μ is the Bohr magneton. Explicit expressions for A_k and B_k are given, e.g., in [1]. Only the resonant terms describing photon decay into two magnons and the inverse problem have been retained in (1), since it is assumed that $\mu h_0 \ll \epsilon_k$. Eliminating the explicit dependence on the time with the aid of the unitary operator $U = \exp[-(i t/2) \sum_k a_k^+ a_k]$, we change over to the equivalent Hamiltonian

$$\tilde{\mathcal{H}} = U^{-1} \mathcal{H} U - i \hbar U^{-1} \frac{\partial U}{\partial t} = \tilde{\mathcal{H}}_0 + \mathcal{H}_{int}, \quad (2)$$

$$\tilde{\mathcal{H}}_0 = \sum_k \left(\epsilon_k - \frac{\hbar \omega}{2} \right) a_k^+ a_k + \frac{1}{2} \sum_k (V_k a_k a_{-k} + V_k^* a_k^+ a_{-k}^+).$$

We retain in the interaction Hamiltonian \mathcal{H}_{int} only the terms of fourth order in the Bose operators, which are due to exchange and contain therefore equal numbers of creation and annihilation operators. The spectrum of an "oscillator" with fixed value of k in the Hamiltonian $\tilde{\mathcal{H}}_0$ is discrete if $|\epsilon_k - \hbar \omega/2| > |V_k|$, and continuous if $|\epsilon_k - \hbar \omega/2| \leq |V_k|$. The k -space region defined by the second inequality is the region of parametric excitation, in which the Heisenberg operators $a_k(t) = \exp[(i/\hbar) \tilde{\mathcal{H}}_0 t] a_k \exp[(-i/\hbar) \tilde{\mathcal{H}}_0 t]$ and $a_k^+(t)$ increase exponentially with time.

The magnon interaction should limit the growth in the region of the parametric excitation. Since magnon pairs with opposite momenta are excited thereby, it should be assumed that the principal role in the limitation should be played by interactions between pairs of the BCS type and the fermi-liquid type

$$\mathcal{H}'_{int} = \frac{1}{2N} \sum_{kk'} (\Phi_{kk'} a_k^+ a_{-k}^+ a_{k'} a_{-k'} + \Psi_{kk'} a_k^+ a_k a_{k'}^+ a_{-k'}). \quad (3)$$

(For explicit expressions for the amplitudes Φ and Ψ see [1]). Such an analysis was made by Zakharov, L'vov, and Starobinets [2, 3]. The remaining interactions lead, as usual, to collision integrals in the kinetic equation.

If the collisions bring the system into a state of thermodynamic equilibrium, then it is possible to introduce a self-consistent field in this state, as a result of which the Hamiltonian (2) is equivalent to

$$\tilde{\mathcal{H}} = \sum_k \left(\epsilon_k - \frac{\hbar \omega}{2} + \Lambda_k \right) a_k^+ a_k + \frac{1}{2} \sum_k (\Delta_k a_k a_{-k} + \Delta_k^* a_k^+ a_{-k}^+) + \mathcal{H}'_{int}, \quad (4)$$

where

$$\Lambda_k = \frac{1}{N} \sum_{k'} \Psi_{kk'} \langle a_k^+ a_{k'} \rangle, \quad \Delta_k = V_k + \frac{1}{N} \sum_{k'} \Phi_{kk'} \langle a_k^+ a_{-k'}^+ \rangle, \quad (5)$$

and the averaging is carried out with the equilibrium density matrix of the system with the Hamiltonian (4). The existence of an equilibrium distribution presupposes, first, the possibility of diagonalizing the Hamiltonian (4). This means that it is possible to introduce with the aid of the u - v transformation new quasiparticles having a dispersion law

$$\tilde{\epsilon}_k = \sqrt{\left(\epsilon_k - \frac{\hbar \omega}{2} + \Lambda_k \right)^2 - |\Delta_k|^2}, \quad (6)$$

where the functions Λ_k and Δ_k , just as in the case of the BCS model for a superconductor [4], is determined from the self-consistency conditions

$$\Lambda_k = \frac{1}{N} \sum_{k'} \Psi_{kk'} \left[\frac{\epsilon_{k'} - \frac{\hbar \omega}{2} + \Lambda_{k'}}{\sqrt{\left(\epsilon_{k'} - \frac{\hbar \omega}{2} + \Lambda_{k'} \right)^2 - |\Delta_{k'}|^2}} \left(\tilde{n}_{k'} + \frac{1}{2} \right) - \frac{1}{2} \right], \quad (7)$$

$$\Delta_k = V_k - \frac{1}{N} \sum_{k'} \Phi_{kk'} \frac{\Delta_{k'}}{\sqrt{\left(\epsilon_{k'} - \frac{\hbar \omega}{2} + \Lambda_{k'} \right)^2 - |\Delta_{k'}|^2}} \left(\tilde{n}_{k'} + \frac{1}{2} \right), \quad (8)$$

where \tilde{n}_k is the equilibrium Bose function for the quasiparticles:

$$\tilde{n}_k = [\exp(\beta \tilde{\epsilon}_k) - 1]^{-1}$$

As seen from (6), the necessary condition for the establishment of an equilibrium state is the inequality

$$|\epsilon_k - \frac{\hbar\omega}{2} + \Lambda_k| > |\Delta_k| \quad (9)$$

for all \vec{k} . In the case of superconductivity, naturally, we do not have the condition (9). The relaxation of the system to the equilibrium distribution n_k is due to the interaction \mathcal{H}_{int} , which leads to a collision integral in the system of new particles with the dispersion law (6).¹⁾

Thus, an equilibrium stationary distribution can set in if Λ_k and Δ_k are solutions of the system of equations (7) and (8) and satisfy the condition (9).

We shall assume that the amplitudes $\Phi_{kk'}$ and $\Psi_{kk'}$ are constant in the region $|\epsilon_k - \hbar\omega/2| \leq |V_k|$ and are equal to zero outside this region²⁾. In addition, we replace for simplicity the function V_k by a constant V . Then, as seen from (7) and (8), the quantities Λ_k and Δ_k are also constant. Since $|V_k| \ll \hbar\omega$, the difference $\epsilon_k - \hbar\omega/2$ can be expanded in the parametric-excitation region and only the terms linear in k need be retained. For zero temperature we then obtain the system of equations

$$\Lambda = \frac{\Psi \left(\frac{\hbar\omega}{2} - \epsilon_0 \right)^{1/2}}{8\pi^2 \theta_c^{3/2}} \left[\sqrt{(\Lambda + |V|)^2 - |\Delta|^2} - \sqrt{(\Lambda - |V|)^2 - |\Delta|^2} - 2|V| \right], \quad (7')$$

$$|\Delta| = |V| - |\Delta| \frac{\Phi \left(\frac{\hbar\omega}{2} - \epsilon_0 \right)^{1/2}}{8\pi^2 \theta_c^{3/2}} \ln \frac{\Lambda + |V| + \sqrt{(\Lambda + |V|)^2 - |\Delta|^2}}{\Lambda - |V| + \sqrt{(\Lambda - |V|)^2 - |\Delta|^2}}, \quad (8')$$

ϵ_0 is the gap in the spin-wave spectrum.

We see first of all from the system (7') and (8') that it has no solutions at $\Psi \leq 0$. Indeed, if $\Psi < 0$ then it follows from the definition (5) of Λ that $\Lambda < 0$. On the other hand, the right-hand side of (7') is then positive. At $\Psi = 0$ ($\Lambda = 0$), the argument of the logarithm in (8') is negative. In the general case, when $\Psi = 0$, $\Phi_{kk'} = \Phi$, and V_k is determined from (1), Eq. (8') likewise has no solution. In the other limiting case when $\Phi = 0$ and $\Psi > 0$, Eq. (7) can be easily shown to have a solution only under the condition

$$\Psi > 8\pi^2 (\sqrt{2} + 1) \frac{\theta_c^{3/2}}{\left(\frac{\hbar\omega}{2} - \epsilon_0 \right)^{1/2}},$$

which cannot be satisfied at any reasonable frequency. Although it is impossible to obtain analogous conditions in explicit form at arbitrary values of Φ and Ψ , the presence of conditions on the interaction amplitudes means that there exist regions of Φ and Ψ where stationary states are impossible.

In such cases, the time behavior of the system is described by the equations for the correlators $\langle a_k^\dagger a_k \rangle$ and $\langle a_k a_{-k} \rangle$. In the simple case when the amplitudes $\Phi_{kk'}$, $\Psi_{kk'}$, and V_k are con-

1) The relaxation terms in the equations for the correlators $\langle a_k^\dagger a_k \rangle$ and $\langle a_k a_{-k} \rangle$ were introduced in [3] without allowance for the spectrum restructuring due to the self-consistent field; the resultant self-consistency conditions are therefore essentially different from (7) and (8).

2) This means that we confine ourselves to the interaction of a magnon pair only in the parametric-excitation region. Outside this region, the contribution of the pair interaction is small to the same degree that the field amplitude is small.

stant, it can be shown that a periodic regime is established in the system, and its period and amplitude depend on the ratio of V to the magnon interaction constants Ψ and Φ .

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EFFECT OF NONMAGNETIC IMPURITIES ON THE CURIE TEMPERATURE OF DILUTE FERROMAGNETIC ALLOYS

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Submitted 3 April 1973

ZhETF Pis. Red. 17, No. 10, 594 - 597 (20 May 1973)

It is shown that introduction of a small concentration of nonmagnetic impurities in dilute ferromagnetic alloys of the PdFe type can change the Curie temperature in a wide range. Measurement of T_C can yield information on the states of the magnetic impurities in the alloy.

The ferromagnetism of dilute alloys of the PdFe type is due to indirect exchange interaction of the localized moments of the impurities via strongly correlated holes in the narrow d-band; this interaction decreases with the distance r between impurities in accordance with the expression $V(r) \sim \exp(-r/R_0)$ [1].

We have shown in [2] that in the case when the average distance between impurities $r_{av} \gg R_0$, the Curie temperature of the alloy is of the order of the interaction energy of impurities the spacing between which is equal to the average distance, and depends on the magnetic-impurity concentration as

$$T_C \sim \exp\left(-\frac{\gamma}{R_0 n^{1/3}}\right),$$

where γ is a coefficient of the order of unity. Such a dependence agrees well with experiment [2, 3].

We show in the present paper that by introducing a small concentration of nonmagnetic impurities in dilute ferromagnetic alloys of the PdFe type it is possible, depending on the properties of these impurities, to decrease as well as to increase T_C in a wide range; in particular, we present an explanation of the results of [3, 4]. In addition, it will be shown that measurement of T_C can yield information on the states of nonmagnetic impurities in the alloy.

In the Hubbard model, the Hamiltonian of the electrons in an alloy $A_{1-c}B_c$ of an almost-magnetic alloy A with nonmagnetic impurities B having a concentration $c \ll 1$ is given by

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2, \quad (1)$$

$$\mathcal{H}_0 = \sum_{\mathbf{k}, \sigma} \epsilon_{\mathbf{k}, \sigma} a_{\mathbf{k}, \sigma}^+ a_{\mathbf{k}, \sigma} + I \sum_{\mathbf{l}} n_{e\mathbf{l}} n_{e+\mathbf{l}}, \quad (2)$$

$$\mathcal{H}_1 = U \sum_{\mathbf{l}_B} \sum_{\mathbf{k}, \mathbf{q}, \sigma} a_{\mathbf{k}+\mathbf{q}, \sigma}^+ a_{\mathbf{k}, \sigma} e^{i\mathbf{q} \cdot \mathbf{r}_e} \quad \mathcal{H}_2 = \Delta \sum_{\mathbf{l}_B} n_{e\mathbf{l}} n_{e+\mathbf{l}}, \quad (3)$$

where $\Delta = I_B - I$, and I_B is the correlation energy in a site occupied by an impurity.

The Hamiltonian \mathcal{H}_2 describes the potential scattering by the impurities, the summation over \mathbf{l}_B is carried out over the site occupied by impurities, and the remaining notation is standard.

We consider first the situation when the perturbation \mathcal{H}_2 can be neglected. In this case the energy of interaction of the magnetic impurities placed in the nonmagnetic alloy $A_{1-c}B_c$ is proportional to the alloys susceptibility averaged over the positions of the nonmagnetic impurities, and the interaction radius is determined by the static homogeneous susceptibility,