Josephson effect and collective excitations in thin-film superconducting bridges

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In superconductors near the critical temperature T_c there can exist weakly damped collective oscillations of the velocity of the condensate p_S/m and of the electric field $\mathbf{E}(x,t)=-\nabla\Phi(x,t)$.[1,2] The effects connected with the onset of such oscillations and with the penetration of the field E to a considerable depth into the superconductor lead, as will be shown in the present communication, to interesting singularities in the behavior of Josephson junctions. This pertains to such types of weakly-coupled superconductors, in which the current density j does not depend on the coordinates (as, for example, in Mercereau–Notarys bridges[3]). In junctions of the point-contact type, these effects are of little significance, since the law governing the field decreases in them as determined by the geometry of the contact.[4]

Let us derive equations describing the Josephson effect in one-dimensional bridges of a dirty superconductor ($T\tau \ll 1$). We use the bridge model analyzed in[5] in the case of zero-gap superconductivity, i.e., we consider a thin superconducting film whose critical temperature $T_c(x)$ depends on the coordinates x along the film $T_c(x)=T^*_c$ at $|x|<d$ and $T_c(x)=T_c>T^*_c$ at $|x|>d$. Assuming that $v_0 \equiv (T-T^*_c)/(T_c-T)\gg 1$ and $v_0 d>\xi(T)$ the critical current $j_c=(3\sqrt{3}/2)j_{GL}[2v_0\sinh(2dv_0/\xi(T))]^{-1}$ of the bridge is exponentially small in comparison with the pair-breaking current j_{GL} of a homogeneous film. The equation for the complex gap $\hat{\Delta}(x)=\Delta(x)\exp[i\chi(x)]$ at $|x|<d$ takes, as in[5], the form

$$\xi^2(T)\frac{\partial^2\hat{\Delta}(x)}{\partial x^2}-v_0^2\hat{\Delta}(x)=0. \qquad (1)$$

The remaining terms (including the terms connected with the anomalous Green's function) can be neglected, since it leads to corrections of the type $(j_c/j_{GL})^2$. At $|x|>d$, the solution for $\hat{\Delta}(x)$, just as in[5], is $\hat{\Delta}(x)=\Delta\tanh[(x+x_0)/\sqrt{2}\,\xi(T)]\times\exp[i\chi(x)]$. Joining together this function and the solution of (1), we get

$$j = j_c \sin\phi(t) + \sigma E(t), \tag{2}$$

where $\phi\equiv 2\chi(d)$ and $E(t)$ is the value of $E(x,t)$ at $x\lesssim d$ (the field in the region of the bridge does not depend on the coordinates). This case differs from that of a superconductor with a gap in that the field E and the gauge-invariant potential $\mu=\frac{1}{2}(\partial x/\partial t)+e\Phi(x,t)$ are described by the equation

$$(\pi/4)(\Delta/T)(\partial/\partial t + \tau_\epsilon^{-1})\mu = -(D/\sigma)\vec{\nabla}\mathrm{j}_N, \tag{3}$$

where $j_N=\sigma E(x,t)$ and τ_ϵ is the energy relaxation time (the connection between $E(x,t)$ and μ is obtained by using the condensate equation of motion $\partial\mathbf{P}_S/\partial t=e\mathbf{E}(x,t)+\nabla\mu$ and the continuity equation of the total current $\mathbf{j}=\mathbf{j}_N+\mathbf{j}_S$. This connection, in terms of the Fourier components, is given by

$$\mathrm{E}_\omega(x) = -(\nabla\mu_\omega)[1+2i\omega T/\pi\Delta^2(x)]^{-1}. \tag{4}$$

With the aid of (3) and (4) we obtain $E_\omega(x)$ and $\mu_\omega(x)$, and establish the connection between the Fourier components of the quantities $E(t)\equiv E(0,t)$ and $\phi(t)$ in (2). At $|k_\omega|\xi(T)\ll 1$ we obtain

$$eE_\omega = \frac{1}{4}(\partial\phi_\omega/\partial t)\left[d + k_\omega^{-1}\left(1+\frac{2i\omega T}{\pi\Delta^2}\right) + \frac{2\sqrt{2}\,i\omega T\,\xi(T)}{\pi\Delta\Delta(d)}\right]^{-1}, \tag{5}$$

where

$$k_\omega^2 = (k'_\omega + ik''_\omega)^2 = (i\omega+\tau_\epsilon^{-1})(1+2i\omega T/\pi\Delta^2)(\pi\Delta/4TD),$$

and

$$\Delta(d) = \Delta/\sqrt{2}\,v_0.$$

The rather complicated form of (5) is due to the fact that at $\omega\gtrsim\tau_\epsilon^{-1}$ amd Δ^2/T the depth of penetration of the field into the superconductor, and consequently also the junction resistance, depend substantially on ω. At low frequencies

$$(\omega \ll \Delta^2/T,\ \tau_\epsilon^{-1})\quad eE(t) = \frac{1}{2}\,\frac{\partial\phi}{\partial t}\,[2(d+l_E)]^{-1}$$

i.e., the connection between E and $\partial\phi/\partial t$ takes the usual form. The quantity $2(d+l_E)$, where $l_E^{+1}=k_{(\omega=0)}$ is the depth to which a low-frequency field penetrates into the superconductor. We note also that the connection between the voltage $V_\omega=2E_\omega(d+k_\omega^{-1})$ and $\partial\phi_\omega/\partial t$ [see (5)] differs from the Josephson relation. However, the latter holds true for the time averaged quantities ($2e\overline{V(t)}=\overline{\partial\phi/\partial t}$).

The current-voltage characteristic (CVC) of the bridge can exhibit hysteresis if the characteristic bridge voltage $V_c=2(j_c/\sigma)(d+l_E)$ exceeds τ_ϵ^{-1} or Δ^2/T. Indeed, let us obtain the form of the CVC at sufficiently large voltage V, when the deviation of the CVC from Ohm's law is small. Then

$$\phi(t) = 2e\bar{V}t + \phi_\sim(t),\ E = \bar{E} + E_\sim,$$

where $\phi_\sim(t)$ and $E_\sim$ are oscillating increments of small amplitude. In first-order approximation we obtain from (2–5)

$$j_0 = j_c V/V_c,\ E_\sim = (j_c/\sigma)\,\mathrm{Im}\exp(2ie\bar{V}t).$$

Substituting $E_\sim$ in (5) we obtain $\phi_\sim$ and the correction to the current j_0:

$$\delta j = \overline{\phi_\sim(t)\cos(2e\bar{V}t)} = j_c(\tilde{V}_c/2\bar{V})$$

$$\times\,\mathrm{Re}\left\{\left(1-4i\frac{eVT}{\pi\Delta^2}\right)(1-4ie\bar{V}\tau_\epsilon/\pi)^{-1}\right\}$$

It is seen that at $\delta j \ll j_0$ we can have $j_0<j_c$, i.e., under the conditions given above the CVC is subject to hysteresis.

We consider now N series-connected closely-lying junctions (in the experiment N reached 2000, and the distance between the junctions was $L\gtrsim 1\ \mu$m [6]). In this case we obtain from (3) and (4) for the field $E^{(n)}$ in the nth junction at $L\gg d$, $L\gg\xi(T)$, and $|k_\omega|\xi(T)\ll 1$

$$E_\omega^{(n)}\left[\mathrm{ch}(k_\omega L)+\frac{2\sqrt{2}k_\omega\xi(T)\,i\omega T\,\mathrm{sh}(k_\omega L)}{(1+2i\omega T/\pi\Delta^2)\pi\Delta\Delta(d)}\right]=\frac{1}{4}\frac{\partial\phi_\omega^{(n)}}{\partial t}\frac{k_\omega\,\mathrm{sh}(k_\omega L)}{1+2i\omega T/\pi\Delta^2}$$

$$+\frac{1}{2}\left(E_\omega^{(n-1)}+E_\omega^{(n+1)}\right). \qquad (6)$$

At $j=0$ and $|\phi_\omega|\ll 1$ it follows from (2) that $E_\omega^{(n)}=-(j_c/\sigma)\phi_\omega^{(n)}$. We seek the solution of (6), just as in the case of a one-dimensional chain of elastically-coupled atoms, by changing over to the collective coordinates $\phi_\omega^{(n)}=\Sigma_q\,\phi_q\exp(inqL)$, where $q=2\pi m/NL$. Then, under the conditions

$$\tau_\epsilon^{-1} \ll \omega, \quad (\Delta^2/T)\ll\omega, \quad \omega\ll\Delta \qquad (7)$$

we obtain the dispersion equation

$$\lambda(\omega/\omega_0)\sin(\omega/\omega_0) = \cos(\omega/\omega_0) - \cos(qL), \qquad (8)$$

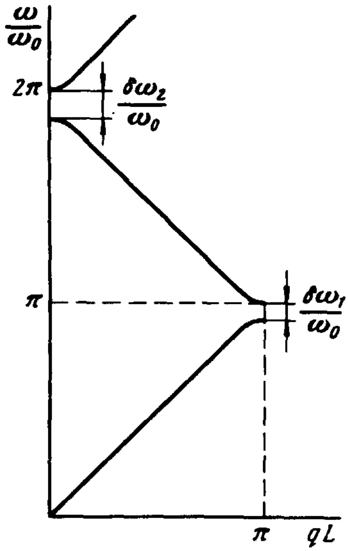

FIG. 1.

where

$$\lambda = \omega_0^2/\omega_j^2\,,\ \omega_J^2 = 16\,DTj_c/\pi L\sigma\Delta,$$

and

$$\omega_0 = (2D\Delta)^{1/2}/L\,.$$

We consider two limiting cases:

a) $\lambda \ll 1$.. The oscillation spectrum is shown in Fig. 1. At $|\omega/\omega_0 - \pi n| > \lambda$ we ha' the acoustic spectrum of the collective oscillations of a homogeneous superconductc $\omega = q\sqrt{2D\Delta}$.[1] As $\omega \to n\pi\omega_0$ the branch is split by the interaction of the junctior and bands of allowed and forbidden frequencies are produced. The distance betwe the bands is $\delta\omega = 2\pi\lambda\omega_0$.

b) $\lambda \gg 1$. In this case, bands are likewise produced. In the first band the spectru is of the same form as the spectrum of acoustic phonons in a crystal: $\omega = \omega_J \sin(qL/$ We note the following important circumstance: the oscillations in the first band a tenuate weakly even if the second condition of (7) is not satisfied. The oscillations the remaining bands are weakly attenuated under the conditions (7). Their spectru is determined by the formula

$$\omega/\omega_0 = \pi n + (\lambda\pi n)^{-1}[1 - (-1)^n \cos(qL)],\quad n = 2, 3, 4\ldots\,.$$

Let us consider two transitions, which we shall use to demonstrate the possibil of observing collective excitations. The equation for $E^{(1,2)}$ is of the same form as ((The difference is that cosh $(k_\omega L)$ in the left-hand side is replaced $\cosh(k_\omega L) + \sinh(k_\omega L)$. It is furthermore easy to verify that weakly-damped oscil tions with frequency ω_J are possible here at $|k_\omega L| \ll 1$, In the presence of the curren this frequency decreases to $\omega_J = \omega_J(1 - j^2/j_c^2)^{1/2}$. We consider now two different ju tions, through which a current j flows such that $j_c^{(1)} < j < j_c^{(2)}$. Proceeding as in t analysis of the hysteresis, we can obtain the correction δj to the CVC near the resonance

$$\delta j = j_c^{(2)}[j_c^{(2)}/j_c^{(1)}]\gamma\omega_J^4(j)\{[(2e\overline{V})^2-\omega_J^2(j)]^2+\gamma^2\omega_J^4(j)\}^{-1},$$

where $\gamma\sim\max[\omega\tau_\epsilon, k_\omega L]_{\omega=2e\overline{V}}$ characterizes the damping. Let now the junctions have critical currents $|j_c^{(1)}-j_c^{(2)}|\ll j_c^{(1)}$. If the current j exceeds the critical points $j_c^{(1,2)}$ then voltages $V^{(1,2)}(t)$ are produced across the junctions and oscillate synchronously with a frequency $2e\overline{V}$ if the distance between them satisfies the relaxation $Lk''_\omega<\mathrm{en}|j_c^{(1)}/(j_c^{(1)}-j_c^{(2)})|$. As the voltage $\overline{V}$ changes, the correction δj to the Ohmic current oscillates with a period $\pi\omega_0/e$ and with an amplitude $\sim j_c^2T[\sigma\Delta^2Lk'_\omega k''_\omega]^{-1}$, if the condition (7) is satisfied.

[1]A. Schmid and G. Schön, Phys. Rev. Lett. **34**, 941 (1975).
[2]S.N. Artemenko and A.L. Volkov, Zh. Eksp. Teor. Fiz. **69**, 1964 (1975) [Sov. Phys. JETP **42**, 896 (1975)].
[3]H.A. Notarys and J.E. Mercereau, J. Appl. Phys. **44**, 1921 (1973).
[4]L.F. Aslamazov and A.I. Larkin, Zh. Eksp. Teor. Fiz. **70**, 1340 (1976) [Sov. Phys. JETP **43**, 698 (1976)].
[5]A.F. Volkov and A.L. Kasatkin, Zh. Eksp. Teor. Fiz. **67**, 1527 (1974) [Sov. Phys. JETP **40**, 760 (1975)].
[6]D.W. Palmer and J.E. Mercereau, Phys. Lett. A **61**, 135 (1977).