Electromagnetic characteristics of leptons and *CP* violation in the $SU(3)\times U(1)$ model

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The electric dipole moments of leptons and the correction, for the weak interaction, to the anomalous magnetic moment of the muon are calculated in the $SU(3)\times U(1)$ model of the weak and electromagnetic interactions.

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1. Various mechanisms of *CP*-nonconservation mechanisms in gauge theories of weak and electromagnetic interactions are widely discussed in the literature of late. In addition to introducing into the Lagrangian a small complex parameter (for example, the imaginary quark-mixing angle),[1] some "natural" mechanisms have been proposed for spontaneous T nonconservation on account of exchange of Higgs bosons in models with $SU(2)$ and $SU(2)\times U(1)$ symmetries.[2]

In the $SU(3)\times U(1)$ model of weak and electromagnetic interactions, recently proposed by Lee and Weinberg,[3] a new method of spontaneous T nonconservation was realized—on account of exchange of gauge $X_{1,2}$ bosons.

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The present article is devoted to a calculation of the electromagnetic characteristics of leptons within the framework of the $SU(3)\times U(1)$ model.[3] It is shown that the electric dipole moments (EDM) of the leptons, predicted by this model, are essentially determined by the contribution of the weak interaction to the anomalous magnetic moments of the leptons, and appropriate estimates are obtained.

2. That part of the Lagrangian which is of interest to us includes the two left-hand and right-hand triplets of leptons

$$(\nu_e, e^-, E^-)_L,\ (\nu_\mu, \mu^-, M^-)_L \text{ and } (E^\circ\ E^-, e^-)_R,\ (M^\circ, M^-, \mu^-)_R$$

which contain besides the usual leptons ($\nu_l, l^-=e^-,\mu^-$) also heavy leptons ($L=E,M$) of the electronic and muonic type, and the octet of vector fields $W\pm$, $U\pm$, Y, Z, and $X_{1,2}$.

The contribution of the weak interaction to the anomalous magnetic moment of the muon is determined in the lowest order by 6 diagrams and is given by

$$a_\mu^w = \frac{Gm_\mu^2}{4\pi^2\sqrt{2}}\left\{\left(\frac{5}{3}\right)_W + \left(\frac{5}{3}\right)_U + \frac{1+l}{4}(1-3w)^2\left(\frac{2}{3}\right)_Z + \frac{1+l}{4l}\left(-\frac{10}{3}\right)_Y + \delta\frac{1+l}{l}\frac{m_M}{m_\mu}\cos\epsilon\,(A)_X - \frac{1+l}{l}(B)_X\right\} \quad (1)$$

where G is the Fermi weak-interaction constant, m_μ and m_M are the masses of μ^- and M^-, l and w are the parameters of the Lagrangian,[3] E is the CP-violating phase shift, the parameter δ determines the splitting of the masses of the X_1 and X_2 bosons, and the indices W, U, Z, Y, and X label the contributions of the corresponding diagrams (we neglect here small quantities of the order of m_μ^2/W_W^2, $m_{M^\circ}^2/W_W^2$ and δ^2). The quantities A and B, which are functions of m_L^2/M_X^2, assume the following forms in two limiting cases of interest:

1) $A=2,\ B=4/3$ at $m_E^2,\ m_M^2 << M_X^2$;

2) $A=1/2,\ B=5/6$ at $m_E^2 << M_X^2,\ m_M^2 >> M_X^2$.

(We note that if we confine ourselves in the Lagrangian of[3] to one scalar octet, then $m_M/m_E=m_\mu/m_e$).

At $l=w=0.2$ [3] we have

$$1)\ a_\mu^w = 2.3\times 10^{-9}(-9.6+12\delta\cos\epsilon\, m_M/m_\mu),$$

$$2)\ a_\mu^w = 2.3\times 10^{-9}(6.6+3\delta\cos\epsilon\, m_M/m_\mu) \quad (2)$$

The available experimental data concerning a_μ [4] enable us to obtain, using (2) and the condition $\epsilon\ll 1$, the following limitations on the parameters of the model:

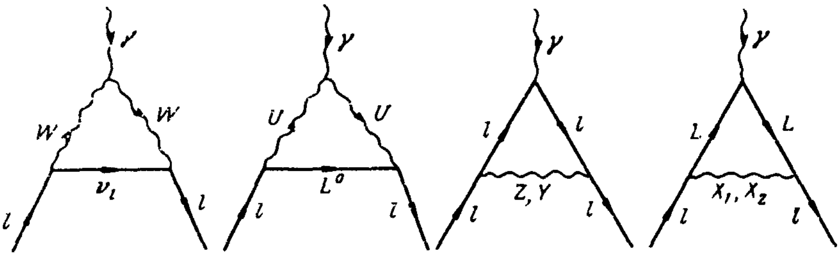

FIG. 1.

$$1)\ \delta m_M / m_\mu \lesssim 2 ; \quad 2)\ \delta m_M / m_\mu \lesssim 6 . \tag{3}$$

It is seen from (2) and (3) that X-boson exchange can make the predominant contribution to a_μ^w.

3. In the considered model, the EDM of the lepton is determined by the contributions of two diagrams with X-boson exchange

$$D_l = e \frac{G m_L}{8\pi^2\sqrt{2}} \sin\epsilon\, \delta \frac{1+l}{l} (A)_X \ . \tag{4}$$

This yields for the EDM of the electron

$$D_e = 2.5 \times 10^{-20} \epsilon\, \delta\, (m_E / \mathrm{GeV})\ e \cdot \mathrm{cm}. \tag{5}$$

Using the inequalitites (3), we obtain electron EDM estimates that do not depend on δ:

$$1)\ D_e \lesssim 5 \times 10^{-21} \epsilon (m_E/m_M)\ e \cdot \mathrm{cm};\ \ 2)\ D_e \lesssim 3 \times 10^{-23} \epsilon\ e \cdot \mathrm{cm}. \tag{6}$$

We note that at reasonable values of ϵ, m_E, and m_M, namely $\epsilon \sim 10^{-3}$ and $m_E \sim m_M$, the EDM of the electron in the Lee–Weinberg model is close to the EDM of the neutron ($D_n \sim 10^{-24} e$ cm [3]).

On the other hand, using the limitation $D_e^{\exp} < 3 \times 10^{-24} e$ cm [1] and the inequalities (5) and (6), we can obtain the following limitations on the parameters of the model

$$\epsilon\, \delta\, m_E \lesssim 10^{-4}\,\mathrm{GeV}\ , \qquad \epsilon\, m_E / m_M \lesssim 0.6 \times 10^{-3}\ .$$

Analogously, from (1) and 3) we obtain for the muon EDM the estimates

$$1)\ D_\mu = 2.6 \times 10^{-21}\ \delta\epsilon\, (m_M / m_\mu)\ e \cdot \mathrm{cm.} < 5 \times 10^{-21} \epsilon\ \ e \cdot \mathrm{cm};$$

$$2)\ D_\mu = 6.5 \times 10^{-22}\ \delta\, \epsilon (m_M/m_\mu)\ e \cdot \mathrm{cm} < 4 \times 10^{-21} \epsilon\ e \cdot \mathrm{cm}. \tag{7}$$

We note that whereas in the former case ($m_M^2 \ll M_X^2$) the electron and muo
EDM can be comparable in magnitude, since $D_\mu / D_e = m_M / m_E$, in the latter case w
have $D_\mu \gg D_e$ (by virtue of the ratio of the masses m_M and m_E).

We have thus shown that the Lee–Weinberg model predicts for the leptons EDM
of the same order of magnitude as for the neutron. Despite the large number of fre
parameters in the model, the muon EDM is determined by a single parameter
namely the *CP*-violating phase shift (7) (with allowance for the restrictions imposed o
the *X*-boson exchange parameters by the data on the anomalous magnetic moment
the muon). Increasing the sensitivity in experiments on the determination of the EDM
of the muon is therefore of exceedingly great importance in verifying the predictions
the Lee–Weinberg model (estimates in[4] give $D_\mu^{\exp} < 10^{-18} e$ cm).

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