Lifetime of strongly excited nuclei

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It was shown in[1], for the case of a single-channel system, that the average li
time $\overline{T}$ of the intermediate system produced in elastic scattering, under conditio
when the average distance D between the resonances is much shorter than their wid
$D \ll \Gamma$, is equal to

$$\overline{T} \approx \frac{\pi h}{D} >> \frac{\pi h}{\Gamma} . \qquad (1$$

A generalization of this result to the case of systems with "n" open channels[2] h
shown that the average lifetime $\overline{T}$ of the compound system produced in the reacti
$i \to j$ is smaller by a factor "n":

$$\overline{T}_{ij}[2] \approx \frac{\pi h}{nD} << \frac{\pi h}{D} \quad \text{if} \quad n >> 1, \qquad (2$$

It will be shown in the present article that at $n>1$ formula (1') is incorrect, and that (1) is valid for any number of open channels. This enables us to use (1) to estimate the lifetimes of strongly excited medium and heavy nuclei having tremendous numbers of decay (for the most part fission) channels.

The formula for the lifetime T_{ij} of an intermediate compound system, in the general case of an arbitrary number of channels, was obtained in[3] for any quantum mechanical system described by the coupled equations

$$T_{ij} = h\,\mathrm{Im}\left(S_{ij}^{-1}\frac{\delta}{\delta E}S_{ij}\right). \tag{2}$$

The subscripts i and j label here the open channels, T_{ij} is the lifetime of the intermediate system in the reaction $i\to j$, S_{ij} is the corresponding element of the S matrix for the given partial state, and $\delta/\delta E$ is an operator defined in[3]. If S_{ij} depends strongly on the energy E, so that

$$\left| E\frac{d}{dE}S_{ij}\right| >> |S_{ij}|, \tag{3}$$

then this operator goes over into the derivative with respect to energy $\delta/\delta E\to d/dE$. This is precisely the case that we shall consider.

To describe the situation wherein the system considered by us has many resonances, we parametrize the S matrix in the form

$$S_{ij} = e^{i(\delta_i+\delta_j)}\left\{\delta_{ij} - \frac{2i\,a_i\,a_j}{\eta + ig}\right\}. \tag{4}$$

Here $\delta_i,\delta_j,a_i,a_j,\eta$ and g are real quantities that depend on E. From the unitarity condition it follows that $g=\Sigma_i a_i^2>0$. The resonances correspond to the zeros of the function η at the energies E_λ near which one can use the expansion $\eta(E)=\eta'(E_L)(E-E_x)$. The resonance width Γ_λ is obviously equal to

$$\Gamma_\lambda = 2g(E_\lambda)/\eta'(E_x) \tag{5}$$

since Γ_λ must be positive, so that $\eta'(E)>0$ for all E. This means that $\eta(E)$ should be a discontinuous function of the type

$$\eta(E) = \gamma\,\mathrm{tg}\frac{E-E_0}{D}\frac{\pi}{2} + \phi(E) \quad \text{or} \quad \eta(E) = \sum_{\lambda=1}^{\lambda_{max}}\frac{C_\lambda}{E_\lambda - E} + \phi(E), \tag{6}$$

where γ and C_λ are positive constants, and $\phi(E)$ is a certain smooth function of the energy. With increasing energy, $\eta(E)$ assumes periodically values $\pm\infty$, and goes in the intervals between the neighboring singularities through the value $\eta=0$. Thus, a resonance is situated between the neighboring singularities $\eta(E_n)=-\infty$ and $\eta(E_n+D)=+\infty$, will be designated D; it is obviously close to the distance between the neighboring resonances.

We shall assume that the principal energy dependence of the S matrix is determined by the quantities η, and a_i, and a_j, and that the phase shifts δ_i and δ_j can be regarded as constants at small changes of the energy $\Delta E \sim D$.

Consider an arbitrary reaction $i\to j$ $(i\neq j)$. Assuming (3) to be valid, we get from (2) and (4)

$$T_{ij} = h\,\mathrm{Im}\left\{\frac{\eta+ig}{2a_i a_j}\left[\frac{2i(a_i a_j)'}{\eta+ig} - \frac{2ia_i a_j}{(\eta+ig)^2}\eta'\right]\right\} = h\frac{\eta' g}{\eta^2+g^2}\,. \tag{7}$$

We average this expression over the energy interval $\Delta E = D$ about some resonance. The upper and lower limits of the integration are chosen to be the points E_n and $E_{n+1}=E_n+D$, such that $\eta(E_n)=-\infty$; $\eta(E_{n+1})=+\infty$. We then readily obtain

$$\overline{T}_{ij} = \frac{1}{D}\int_{E_n}^{E_n+1} dE\,\frac{hg\eta'}{\eta^2+g^2} = \frac{hg}{D}\int_{-\infty}^{\infty}\frac{d\eta}{\eta^2+g^2} = \frac{h\pi}{D}\,. \tag{8}$$

Averaging over a larger energy interval yields the same result, since this interval can be broken up into smaller ones about each resonance, analogous to those used in the derivation of (8).

We estimate $\overline{T}_{ij}$ by using the standard formula for the level density of a nucleus[4] consisting of A nucleons and excited to the characteristic value $E^*=10$ MeV

$$D = \frac{A^{1/4}}{0{,}012}\exp(-1{,}68A^{1/2})\,\text{MeV}. \tag{9}$$

We easily obtain

A	50	100	150	200	250
D, MeV	10^{-3}	10^{-5}	10^{-7}	10^{-8}	10^{-9}
T, sec	10^{-18}	10^{-16}	10^{-14}	10^{-13}	10^{-12}

The result (longer lifetimes) is unexpected, but it can apparently be verified experimentally by using either the "shadow" method or by measuring the probabilities of γ radiation from a nucleus excited to an energy E^*.

In connection with these estimates it should be noted that they cannot be used at too large excitation energies, when the resonant structure of the S matrix vanishes and the formula for the level density [see (9)], obtained assuming an infinite Fermi system or else assuming a finite system incapable of decay, becomes meaningless. In this respect, the second formula of (6), where the number of resonances is limited, is preferable to the first, since it takes into account the fact that at $E > E_{\lambda\max}$ the function η ceases to be rapidly alternating and becomes smooth. In this case T decreases sharply to a value of nuclear order ($T \sim 10^{-22}$ sec).

The incorrect result (1′) obtained in[2] is due to the fact that the lifetime was calculated in that paper not directly, but starting in fact from the value of an integral of $|\psi_i|^2$ over the volume of the nucleus[5] (ψ_i is the total wave function of the system, corresponding to the ith input channel). This integral can be easily calculated [see (4) and formula (12) of[6]] and turns out to be equal to the product of the average lifetime T_{ij} by the relative probability of production of a compound nucleus by the i channel: $\overline{T}_{ij}^{[2]} = T_{ij}\Gamma_i/\Gamma$; ($\Gamma_i$ and Γ are the partial and total widths). It is the factor Γ_i/Γ that describes this probability which leads to the number η of open channels in the denominator of (1′). This circumstance was not taken into account in[2]. In the single-channel case, the relative probability is equal to unity, and this method yields the correct result.

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