Giant oscillations of thermal conductivity of the intermediate state of a superconductor

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(Submitted 21 November 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **27**, No. 3, 152–155 (5 February 1978)

A theoretical investigation was made of the oscillatory thermomagnetic effects in the intermediate state of superconductors with open Fermi surfaces. Giant oscillations of the coefficient of thermal conductivity, due to the open electron trajectories, were observed.

PACS numbers: 74.30.Ek, 74.40.+k, 74.55.+h, 71.25.Pi

At low temperatures $T \ll T_c$, owing to the Andreev reflection[1] of the normal electronic excitations from the interface between the normal and superconducting phases, the thermal conductivity of the intermediate state (IS) of a type-I superconductor with mean free path l exceeding the Larmor radius r_H in a critical magnetic field H_c reveals a nonmonotonic dependence on the thickness L of the normal layers; this dependence was predicted in[2,3] within the framework of the model of a metal with an isotropic dispersion law. The present investigation concerned the role of anisotropy and of topological singularities of the Fermi surface in the onset of oscillatory thermomagnetic effects in IS of a superconducting metal with a complicated dispersion law $\epsilon(p)$.

Owing to the vanishingly small thermal conductivity of the superconducting layers at $T \ll T_c$, the heat flux q_α averaged over the sample section is directed along the layers (in the yz plane) and is proportional to the concentration η of the normal phase:

$$q_\alpha = -\kappa_{\alpha\beta} \nabla_\beta T = 2\eta \int_0^L \frac{dx}{L} \int \frac{d^3p}{(2\pi)^3} v_\alpha \frac{\xi^2}{T} \frac{\partial f^{(0)}}{\partial \xi} \chi(\mathbf{n}, x). \tag{1}$$

Here $\xi = \epsilon(\mathbf{p}) - \epsilon_F$ is the energy, $\mathbf{p} = \mathbf{n}|\mathbf{p}|$ is the momentum, and $\mathbf{v} = \partial\epsilon(\mathbf{p})/\partial\mathbf{p}$ is the velocity of the electrons, $\kappa_{\alpha\beta}$ is the tensor of the longitudinal thermal conductivity of the IS, $(\xi/T)(\partial f^{(0)}/\partial\xi)\chi$ is a small increment to the local-equilibrium Fermi distribution function $f^{(0)}(\xi)$ and satisfies the linearized Boltzmann kinetic equation with the boundary condition[3] $\chi(\mathbf{n}, x_0) = \chi(-\mathbf{n}, x_0)$ corresponding to the Andreev reflection from the boundaries $x_0 = 0$ and $x_0 = L$ of the normal layer:

$$\chi(t, p_z, x) = \nabla T \int_{-\infty}^{t} dt' \mathbf{v}(t') \exp\left(\frac{t' - t}{\tau}\right)\left(-1\right)^{\left[\phi(t,t') - \frac{x}{L}\right]},$$

$$\phi(t, t') = \frac{p_y(t') - p_y(t)}{eH_c L} \tag{2}$$

t is the time of motion of the electron over the Fermi-surface section p_z=const in a magnetic field $\mathbf{H}=(0,0,H_c)$; τ is the relaxation time, and $[a]$ is the integer part of a. According to (2), the motion of the electron in momentum space is accompanied by a reversal of the sign of the velocity (the conversion of the electronic excitation into hole excitation[1,3]) upon intersection with the lines

$$p_y = p_y(t) + eH_c x + neH_c L \qquad (n = 0, \pm 1, \pm 2, \ldots), \tag{3}$$

that form a periodic structure in momentum space, with a period eH_cL. Substituting (2) in (1), it is easy to verify that the nonmonotonic dependence of the thermal conductivity on the parameters of the laminar structure of the IS, due to the resonant character of the interaction of the electrons with the periodic structure (3), is perfectly analogous to the magnetoacoustic resonant phenomena[4,5] that occur in a normal metal under conditions of spatial periodicity produced by a sound wave. Owing to a small thickness of the normal layers $L \ll r_H$, the phase $\phi(t,t')$ in (2) is large ($\sim r_H/L$) and the main contribution to the heat transport along the layers is made by the vicinity of the stationary points $\dot{p}_y(t_\mu)=-eH_c v_x(t_\mu)\equiv -eH_c v_{x\mu}=0$, i.e., by the trajectory sections on the "strip" $v_x=0$ of the Fermi surface, where the electron moves practically parallel to the phase-separation boundaries:

$$\kappa_{\alpha\beta} = \frac{4}{3\pi^3}\eta TL \int dp_z \sum_{0<t_\mu\leqslant T_H} \sum_{-\infty<t_\nu\leqslant t_\mu} \frac{v_{\alpha\mu} v_{\beta\nu}}{|v'_{x\mu} v'_{x\nu} v_{y\mu} v_{y\nu}|^{1/2}}$$

$$\times \exp\left(\frac{t_\nu - t_\mu}{\tau}\right) \sum_{n=0}^{\infty} (2n+1)^{-3} \cos\left(\pi(2n+1)\frac{D_{\mu\nu}}{eH_c L} - \frac{\pi}{4}(s_\mu - s_\nu)\right),$$

$$s_\mu = \operatorname{sign} v_{x\mu}.$$

Here T_H is the cyclotron period, and $v'_x \equiv \partial v_x/\partial p_x$, $D_{\mu\nu}=p_{y\mu}-p_{y\nu}$ is the diameter of the Fermi surface in the direction of the p_y axis.

To investigate the singularities of the thermomagnetic effects connected with the concrete structure of the electronic spectrum, we consider a superconducting metal having a Fermi surface of the "corrugated cylinder" type, with an axis oriented perpendicular to the magnetic field in such a way that the layer of the open trajectories intersect the strip $v_x=0$.

In this case, the terms with $\mu=\nu$, in the region of values of p_z corresponding to closed trajectories, cause a "smooth" dependence of $\kappa_{\alpha\beta}$ on the parameters of the IS structure:

$$\kappa^{(0)}_{\alpha\beta} = \frac{7\zeta(3)}{6\pi^3}\eta\tau TL \int \frac{dp_z}{T_H(p_z)} \sum_{0<t_\mu\leqslant T_H} \frac{v_{\alpha\mu} v_{\beta\mu}}{|v'_{x\mu} v_{y\mu}|} \sim \eta \frac{L}{r_H}\kappa_n, \tag{4}$$

where κ_n is the thermal conductivity of an infinite normal metal at $H=0$. The terms with $\mu\neq\nu$ describe an oscillatory effect with a period

$$\Delta(L^{-1})=\frac{2eH_c}{D_{\mu\nu}}, \tag{5}$$

i.e., a peculiar geometric resonance between the diameter $D_{\mu\nu}$ of the Fermi surface and the thickness eH_cL of the normal layer in momentum space, contributions to which are made in the general case by sections with extremal values of $D_{\mu\nu}$, as well as by a self-intersecting and other singular sections of the Fermi surface. The waveform of the oscillations is quite close to harmonic, and the dependence of the amplitude of the effect on L is described by the power-law function $(L/r_H)^\alpha$ (or $L/r_H)^\alpha \ln^{-1}(r_H/L)$– for self-intersecting sections) with an exponent α that depends on the character of the corresponding singularities of the Fermi surface.

A substantial circumstance in the study of the effects due to the presence of open trajectories, is that the increment of the momentum component $p_y(t)$ over the period T_H of electron motion over the open trajectory is equal to the period of the reciprocal lattice B_y in the direction of the p_y axis, and is consequently the same for all the electrons on the open trajectories. As a result, the contribution of the latter to the thermal conductivity

$$\kappa_{\alpha\beta}=\frac{4}{3\pi^3}\eta TL\int dp_z \sum_{0<t_\mu\leq T_H}\frac{v_{\alpha\mu}v_{\beta\mu}}{|\dot{v}_{x\mu}v_{y\mu}|}\sum_{n=0}^{\infty}(2n+1)^{-3}$$

$$\times\operatorname{Re}\left\{1-\exp\left(\frac{\pi i B_y}{eH_cL}(2n+1)-\frac{T_H}{\tau}\right)\right\}^{-1} \tag{6}$$

contains the characteristic "resonance denominator" which is known from the theory of magnetoacoustic resonance[5] and describes giant oscillations of the thermal conductivity of the IS against the background of the smooth contribution of the closed trajectories (4), modulated by the weak quasiharmonic oscillations (5). According to (6), the resonance maxima of the thermal conductivity, with amplitude comparable with $\kappa_{\alpha\beta}^{(0)}$, are localized near values of L corresponding to the condition that the period of the reciprocal lattice be commensurate with the thickness of the normal layer in momentum space:

$$L_{n,m}=\frac{2n+1}{2m}\frac{B_y}{eH_c}\qquad\left(\begin{matrix}n=0,1,2,\ldots\\ m=\pm1,\pm2)\ldots\end{matrix}\right). \tag{7}$$

A given number n corresponds to a series of maxima with different m and with identical relative amplitude that decreases with n like $(2n+1)^{-3}$. The period of the giant oscillations, i.e., the distance between the neighboring maxima of the nth series $L_{n,m}-L_{n,m+1}\sim L_{n,m}/m$ greatly exceeds their width $(\Delta L)_{n,m}\sim r_HL_{n,m}/ml$. Estimates show that it is possible to observe in experiment the maxima belonging to the first two series ($n=0$ and 1) with numbers $m \sim r_H/L$.

It is useful to note another aspect of the physical nature of the oscillatory effects in the IS, an aspect connected with the singularities of the motion of the electronic excitations in coordinate space along the normal layers over a complicated trajectory consisting of segments of the section $p_z=\text{const}$ of the Fermi surface.[2,3] Analysis shows that a closed trajectory in momentum space corresponds to a slow directional drift along the layers, with a velocity (and hence with a heat-flux intensity) that depends nonmonotonically on L. At the same time, the infinite motion over the open trajectory in momentum space, under conditions of Andreev reflection, has generally speaking two noncommensurate periods, B_y and eH_cL. In this case the drift of the electronic excitation in coordinate space has a random-walk character with an average velocity different from zero only at a definite rational relation between B_y and eH_cL [Eq. (7)], corresponding to maximum heat flow in the normal layer.

We indicate in conclusion that investigation of the thermal conductivity of the IS can serve as a method of reconstructing the shape and the topology of the Fermi surface of a metal from the known dependence of L on the external magnetic field H_0. Measurement of the period of the quasiharmonic oscillations (5) at different orientations of H_0 relative to the crystallographic axes of the sample yields information on the diameters of the Fermi surface, and an investigation of the dependence of the amplitude of the oscilations on L yields information on the character of the corresponding singularities of the electronic spectrum. The appearance of giant oscillations attests to the presence of open trajectories and makes it possible to assess the topological properties of the Fermi surface.

The author thanks V.P. Galaĭko for useful discussions of the results.

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