Anomalies of the elastic moduli of a metal under the conditions of the Shoenberg effect

V. N. Bagaev, V. I. Okulov, and E. A. Pamyatnykh

Institute of Metal Physics, Urals Scientific Center, USSR Academy of Sciences Urals State University

(Submitted 28 November 1977)

Pis'ma Zh. Eksp. Teor. Fiz. **27**, No. 3, 156–158 (5 February 1978)

It is shown that when the stability of the magnetization of an electron gas at the peaks of the quantum oscillations of the differential susceptibility is lost, magnetostriction can give rise to a sharp decrease of the elastic moduli of a metal.

PACS numbers: 75.80.+q

The amplitude of the quantum oscillations of the magnetic susceptibility χ of a metal in the de Haas–van Alphen effect can reach such large values that the stability condition $1-4\pi\chi>0$ is violated at the maxima and there appear jumps of the magnetization as functions of the magnetic field as well as "diamagnetic" domains.[1,2] It is shown in the present communication that near the instability points, owing to the coupling of the magnetization of the electrons with the lattice, anomalies can appear in the elastic moduli of the metal. Although the influence of magnetostriction on the quantum oscillations of the elastic moduli has already been considered in[3], the approach used there did not make it possible to detect the effect predicted here.

This effect seems from the fact that in a quantizing field B the additional inhomogeneous field $b(r)$ leads to a redistribution of the electron density, i.e., to the appearance of an increment $\delta N(r)=(\partial N/\partial B)b(r)=(-\partial N/\partial\zeta)(\partial\zeta/\partial B)b(r)$ to the initial concentration N (ζ is the chemical potential of the electrons). On the other hand, the local charge densities of the electrons and of the lattice are connected by the electroneutrality condition

$$\delta N(r) - Q\,\mathrm{div}\,u(r) = 0, \tag{1}$$

where $Q=-eN$ is the charge density of the undeformed lattice and $u(r)$ is the displacement vector. Therefore the field $b(r)$ leads to deformation of the metal and consequently the magnetic ordering of the system of electrons in the Shoenberg effect should be accompanied by a restructuring of the lattice.

To obtain a formula that describes the oscillations of the tensor of the elastic moduli, we assume a simplified model that takes no account of the deformation mechanism of the interaction of the electrons with the lattice; a more complete analysis does not lead to a qualitative change of the main conclusions. In such a model, the electronic part of the elastic force is due to the self-consistent field produced by the strain. The potential ϕ of the field is determined by Eq. (1), in which it is necessary to take into account the corresponding change δN, putting

$$\delta N = -\frac{\partial N}{\partial \zeta} e\phi + \frac{\partial N}{\partial B} b \equiv -N_\zeta \left(e\phi + \frac{\partial \zeta}{\partial B} b \right), \tag{2}$$

where the field b is now determined from the equations

$$\text{rot } b = 4\pi \text{ rot } \delta M = 4\pi \text{ rot} \left\{ -\frac{\partial M}{\partial \zeta} e\phi + \frac{\partial M}{\partial B} b \right\}; \qquad \text{div } b = 0, \tag{3}$$

M is the electron magnetization. By finding the potential from the system (1)–(3), we get the force acting on the lattice

$$F = -Q \nabla \phi = eN \nabla \phi = (N^2/N_\zeta)\Delta u - N\nabla \left(\frac{\partial \zeta}{\partial B} b \right), \tag{4}$$

which determines the electronic part of the elastic moduli. The first term in the right-hand side of (4) is connected with the usual contribution of the electrons to the compression modulus, while the second term is due to the change of the magnetization M upon deformation. By eliminating the potential ϕ, we can reduce the equation for the field b to the form

$$\text{rot} \{ (1 - 4\pi \chi) b \} = -4\pi \text{ rot} \left\{ \frac{\partial M}{\partial \zeta} \frac{N}{N_\zeta} \text{div } u \right\}, \tag{5}$$

where $\chi = \partial M/\partial B + (\partial M/\partial \zeta)(\partial \zeta/\partial B) \equiv \chi_B + \chi_\zeta$ is the susceptibility of the electrons with allowance for the change of the chemical potential in the magnetic field. It follows from this equation that at small values of the susceptibility ($\chi \ll 1/4\pi$) the magnetostriction contribution to the force F is relatively small because of the smallness of $4\pi\chi_\zeta$ in comparison with unity; the main contribution to the quantum oscillations of the elastic moduli is made in this case by the state density N_ζ. On the other hand, under the conditions of the Shoenberg effect, the field b increases near the points of the magnetization instability (when $4\pi\chi \approx 1$), thus ensuring "softening" of the lattice.

For a metal with a spherical Fermi surface, the anomaly manifests itself in quantum oscillations of the velocity s of the longitudinal sound propagating across the field B. With the aid of (4) and (5) we obtain

$$s^2 = \frac{N^2}{\rho_m N_\zeta} \left(1 + \frac{4\pi \chi_\zeta}{1 - 4\pi \chi} \right) \approx \frac{N^2}{\rho_m g} \left[1 - \frac{\Delta}{\gamma(1 - 4\pi \kappa \gamma^3 \Delta)} \right], \tag{6}$$

where ρ_m is the mass density of the lattice, and the oscillating parts of the quantities N_ζ, χ_ζ, and χ were separated with the aid of an asymptotic expansion in the ratio of the cyclotron quantum $\hbar\Omega$ to the Fermi energy E_F:

$$N_\zeta = g\left(1 + \frac{\Delta}{\gamma} \right); \qquad \chi = \kappa \gamma^3 \Delta; \qquad \chi_\zeta = -\kappa \gamma^2 \Delta^2; \tag{7}$$

$$\Delta = -2\theta e^{-\theta}\cos\pi\left(\gamma^2 - \frac{1}{4}\right)\cos\pi\frac{\Omega_0}{\Omega}; \qquad (8)$$

$\gamma=(2E_F/\hbar\Omega)^{1/2}$, $\hbar\Omega_0$ is the spin-splitting energy, $\theta=2\pi^2T/\hbar\Omega$, and T is the temperature in energy units. Formula (8) for the oscillating function Δ was assumed under the assumption that $\theta\sim 1$. The instability of the magnetization appears at the points for which $4\pi\kappa\gamma^3\Delta\approx 1$; when these points are approached, the electronic part of the sound velocity s tends to zero in accordance with (6).

The results presented above can be obtained also by purely thermodynamic means, if the stress tensor is obtained by differentiating the thermodynamic potential of the electrons in a magnetic field with respect to the strain tensor. In this case it is necessary to use the thermodynamic potential in terms of the variables ζ and M (with account taken of the term $2\pi M^2$, see[1,2]), and with allowance for the dependence of the quantities ζ and B on the strain tensor (at constant H), as well as for the $\zeta(B)$ dependence.

We note in conclusion that by virtue of the inequality $\chi_\zeta \ll \chi$ [see (7)] it is possible to observe the predicted anomaly in the dependence of the elastic moduli on the magnetic field in a region close enough to the instability points.

[1]D. Shoenberg, Philos, Trans. R. Soc. London Ser. A **255**, 85 (1962); A.B. Pippard, Proc. R. Soc. London Ser. A **272**, 192 (1963); J.H. Condon, Phys. Rev. **145**, 526 (1965).
[2]I.M. Lifshitz, M.Ya. Azbel', and M.I. Kaganoc, Elektronnaya teoriya metallov (Electron Theory of Metals), Nauka, 1971, §20.
[3]L.R. Testardi and J.H. Condon, Phys. Rev. B **1**, 3928 (1970).