Propagation of phonon pulses in the regime of spontaneous phonon decay

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We consider ballistic and diffuse propagation of phonon pulses accompanied by degradation of the frequency spectrum in the spontaneous decay of the phonons.

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It is customarily assumed that there are three regimes of phonon-pulse propagation[1]: ballistic (B), when the phonons are not scattered at all on the path L from the source to the detector; diffusion (D), when the phonon is scattered repeatedly on this path; and hydrodynamic, when process of establishment of local equilibrium of the phonon gas prevail. All these regimes have a common feature–the characteristic frequency of the nonequilibrium phonons remains unchanged in the course of propagation.

Yet this is not the case at a sufficiently high radiator "temperature" $\overline{T}$. With increasing characteristic frequency of the injected phonons, $\omega_0 \approx \overline{T}$, the mean free path l with respect to the phonon-phonon interactions decreases rapidly and the case $l < L$ is realized. On the other hand, if ω_0 greatly exceeds the helium-bath temperature, and the occupation numbers of the injected phonons are $n(\omega_0) \ll 1$, then spontaneous multiplication of the phonons predominates among the phonon-phonon interactions (if $n(\omega_0) \gtrsim 1$, then coalescence of phonons is also significant). Therefore, when a phonon pulse propagates, the average phonon frequency decreases and the propagation regime becomes quite unique. This regime was apparently observed in the propagation of phonon pulses in anthracene.[2]

Consider the propagation of phonons under conditions when two processes are significant: decay, with a time $\tau(\omega) = \tau_0(\omega/\omega_0)^{-5}$,[3] and elastic scattering, with a time $\tau^*(\omega) = \tau_0^*(\omega/\omega_0)^{-4}$.[3] Assume that phonons are instantaneously injected into a semi-infinite crystal through a certain point on the boundary. A detector is located at a distance L from the source. Which phonons will reach this point? What is their flux? The solution of the kinetic equation, even under highly simplifying assumptions, cannot be obtained. We shall therefore use an illustrative procedure that provides the necessary order-of-magnitude estimates.

We visualize the development of the phonon distribution as a succession of generations. The first generation are the injected phonons. Their spontaneous division yields a generation with a characteristics frequency $\omega_0/2$, followed by a generation with $\omega_0/2^2$ and so on. The lifetime of the generation of frequency ω is $\tau(\omega)$; it increases rapidly as the generations succeed each other, so that the time when the generation with frequency ω appears is $t = \tau(\omega)$. This relation can also be understood differently: it determines the characteristic frequency $\omega(t)$ of the phonons that survive up to the instant t.

Each generation with frequency ω displaces the spatial front of the distribution $R(t)$ by a certain distance $\Delta R(\omega)$. The value of $\Delta R(\omega)$ depends on the relation between $\tau(\omega)$ and $\tau^*(\omega)$. If $\tau(\omega) \gg \tau^*(\omega)$, then the displacement of the generation is of diffuse character: $\Delta R(\omega) \approx v[\tau(\omega)\tau^*(\omega)]^{1/2}$, where v is the speed of sound. If $\tau(\omega) \ll \tau^*(\omega)$, then the generation is displaced ballistically: $\Delta R(\omega) \approx v\tau(\omega)$. In either case $\Delta R(\omega)$ increases rapidly as the generations succeed one another, so that the position of the front is determined by the displacement of the last generation $R(t) \approx \Delta R[\omega(t)]$.

Behind the front, the total energy of the injected phonons E_0 is uniformly distributed, and therefore the energy density at a certain point behind the front is $\epsilon(t) \approx E_0/R(t)^3$. More readily significant in detection is the energy flux density $p = \alpha\epsilon v$, where α is the degree of anisotropy of the phonon momentum distribution. If the displacement of the generation is diffuse, then $\alpha(\omega) \approx v\tau^*(\omega)/\Delta R(\omega) \approx [\tau^*(\omega)/\tau(\omega)]^{1/2} \ll 1$, if it is ballistic, then $\alpha(\omega) \approx 1$.

The actual evolution of the distribution depends on the ratio τ_0^*/τ_0. If $\tau_0^* \ll \tau_0$, then scattering prevails over decay for all generations. The phonon propagation constitutes diffusion accompanied by simultaneous decay. This regime will be called quasidiffuse (QD). In this regime, the time of arrival of the front at the detector is $\tau_A \approx \tau_B^{10/9}\tau_0^{4/9}/\tau_0^{*5/9}$, where $\tau_B = L/v$ is the time of reaching the regime B. It is important to note that $\tau_A \propto L^{10/9}$ rather than $\propto L^2$, as in the D regime. This much weaker dependence of τ_A on L is due to the fact that as the phonons advance the scattering decreases and the diffusion coefficient increases. It is also obvious that $\tau_A \gg \tau_B$, although the delay of the pulse in the QD regime is not so large as in the D regime. The energy flux density at the instant of the arrival of the pulse is $p_A \approx (E_0 v/L^3)(\tau_B/\tau_A)$ and the characteristic frequency of the arriving phonons is $\omega_A \approx \omega_0(\tau_A/\tau_0)^{-1/5}$. At times $t \gg \tau_A$, the frequency decreases like $t^{-1/5}$ and the flux like $t^{-14/5}$.

We proceed now to the case $\tau_0^* \gg \tau_0$. Then, for the first generations, decay prevails over scattering. The propagation of the phonons us ballistic and is accompanied by decay. This regime will be called quasiballistic (QB). At the generation with frequency $\overline{\omega} = \omega_0(\tau_0/\tau_0^*)$, for which $\tau(\overline{\omega}) = \tau^*(\overline{\omega}) \equiv \overline{t}$, the regime changes to QD. This takes place at times of the order of $\overline{t}$, when the front occupies the position $\overline{R} \approx v\overline{t}$. If the detector is in a remote position, $L \gg \overline{R}$, then the generation $\overline{\omega}$ can be regarded as an effective source and we can use the formulas for the QD regime, in which we substitute the time $\overline{t}$ in place of τ_0 and τ_0^*; it is easily seen that the formulas assume in this case their previous form. If $L \ll \overline{R}$, on the other hand, then at times $t < \overline{t}$ the detector signal will be different. Now the time of arrival is $\tau_A \approx \tau_B$ and $p_A \approx E_0 v/L^3$; at $t \gg \tau_A$ we have $p \propto t^{-3}$. The law governing the frequency of the arriving phonons does not depend on the propagation regime.

In the QB regime the time of arrival is of the same order as in the B regime. The point is that the total momentum of the phonons is conserved in the decay; this ensures preservation of the high anisotropy of the momentum distribution, which in the case of the injected phonons is of the order of unity. By the same token, a high propagation velocity is ensured, On the other hand, there is a long tail behind the front in the QB regime, but not in the B regime.

The signal observed from a strongly "heated" source has just this form[4]. It must be indicated, to be sure, that it is quite difficult to distinguish between the QB and QD regimes, since actually the times τ_A can differ not very strongly, and the "tails" are practically equal.

On going to a planar geometry (a source in the form of a half-plane), only the formulas for the flux change; instead of $E_0 v/L^3$ we must write $E_0 v/AL$, where A is the area of the source, and the laws governing the decrease are $p \propto t^{-1}$ (QD) and $p \propto t^{-11/10}$ (QB).

We note in conclusion that the decrease of the characteristic frequency of the phonons in the course of propagation must be taken into account in the interpretation of experiments on the action of phonon pulses on electron-hole drops, in as much as according to[5] the force acting on the drop in proportional not to p but to ω_p.

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