Nucleon and antinucleon annihilation cross section at nonrelativistic energies

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(Submitted 29 December 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **27**, No. 3, 197–201 (5 February 1978)

It is shown for the low-energy $N\bar{N}$ interaction, within the framework of the coupled-channel scheme, that in the case of a small annihilation radius $r_a \sim m_N^{-1} \sim 0.2$ F and strong attraction it is possible to obtain a consistent description of the small widths of the quasinuclear mesons ($\Gamma \sim 10$–30 MeV) and the large annihilation cross sections. The slow $N\bar{N}$ annihilation cross section deviates in this case from the $1/v$ law. The character of the deviation depends on the potential $N\bar{N}$ interaction.

PACS numbers: 13.75.Cs, 11.80.Gw

A simple nonrelativistic model of two coupled channels h and l was proposed recently[1] for the study of the effect of annihilation on the position and width of the coupled state of a nucleon and antinucleon. The channel h corresponds to heavy particles with mass m and is the analog of the $N\bar{N}$ system. An attraction potential $V_h(r)$ acts between $\bar{N}$ and N and leads to formation of quasinuclear resonances.[2] The channel l is the analog of the boson annihilation channel and describes light particles with mass $\mu < m$. Transitions between the channels are effected by a short-range potential V_{lh}, chosen for simplicity in the separable form $V_{lh} = \lambda g(r) g(r')$, where $g(r) = \exp[-r/r_a]/r$. It is shown in[1] that at any constant λ the widths and shifts of the resonances due to annihilation remain small, on the order of 10–30 MeV provided that the ratio of the annihilation-potential radius r_a to the bound-state radius R is small ($r_a/R \sim 10^{-1}$).

The existing experimental data on the $N\bar{N}$ interaction at low energies ($E_{\rm coll} \gtrsim 25$ MeV) indicate that the annihilation cross section $(\sigma v)_{\rm an} \approx 26$ mb is not much less than the cross section for elastic scattering $\sigma_{el} \sim 100$ mb (we use a system of units $c = \hbar = 1$). The question arises whether it is possible to obtain at a small annihilation radius a large annihilation cross section comparable with the elastic-scattering cross section.

This question was answered in the affirmative by I.S. Shapiro.[3] The $p\bar{p}$ annihilation cross section was defined by the formula $(\sigma v)_{\rm an} = (\bar{\sigma} v)_{\rm an} |\psi_v(0)|^2$, in which the quantity $(\sigma v)_{\rm an}$ could be obtained from dimensionality considerations: $(\bar{\sigma} v)_{\rm an} \sim 2\pi r_a^2$, and the square of the modulus of the wave function $\psi_v(0)$ of the continuous spectrum at zero determined the amplification coefficient of the annihilation cross section.

It is important to obtain expressions for the annihilation cross section in an exactly solvable model of coupled channels. To find the amplitudes it is convenient to write the Schrödinger equation in integral form

$$\hat{\Psi} = \hat{\Psi}_0 + \int \hat{G} \hat{V} \hat{\Psi}, \tag{1}$$

where $\hat{\Psi}$ is the column $\begin{pmatrix}\psi_h\\ \psi_l\end{pmatrix}$; the Green's function $\hat{G}$ is a diagonal matrix: $G=\begin{pmatrix}G_h 0\\ 0 G_l\end{pmatrix}$, with $G_h=[E-p^2/m-V_h(r)]^{-1}$, $G_l=[E+2(m-\mu)-k^2/\mu]^{-1}$, and the interaction $\hat{V}$ takes into account the mixing of the channels: $\hat{V}=(0/V_{lh})(V_{hl}/0)$. To find the elastic-scattering and annihilation amplitudes it is necessary to find the solution of Eq. (1) in the form of a "plane + diverging waves" in the h channel and "diverging wave" in the l channel. To this end we choose $\hat{\Psi}_0=\begin{pmatrix}\psi_{0h}\\ 0\end{pmatrix}$, where ψ_{0h} is the solution in the form in channel h without allowance for the V_{hl} interaction. Taking into account the separability of V_{hl}, we easily write down the solution of Eq. (1).

$$\begin{cases} |\psi_h\rangle = |\psi_{h_0}\rangle + \lambda^2 G_h|g\rangle \dfrac{\langle g|G_l|g\rangle\langle g|\psi_{h_0}\rangle}{1-\lambda^2\langle g|G_h|g\rangle\langle g|G_l|g\rangle} \\ |\psi_l\rangle = \lambda G_l|g\rangle \dfrac{\langle g|\psi_{h_0}\rangle}{1-\lambda^2\langle g|G_h|g\rangle\langle g|G_l|g\rangle} \end{cases} \qquad (2)$$

The coefficient of the diverging wave in $|\psi_l\rangle$ determines the amplitude f_{hl} of the transition of heavy particles into light ones. The expression (2) for $|\psi_l\rangle$ contains as a factor the matrix element $\langle g|\psi_{h0}\rangle$. This quantity can be easily estimated by taking into account the smallness of the ratio r_a/R:

$$\langle g|\psi_{h_0}\rangle \approx \psi_v^{(h_0)}(0)\frac{4\pi}{\beta^2+p^2}, \qquad (3)$$

where $p=mv$ is the momentum of the h particles[1], and $\beta=r_a^{-1}$. For $p\ll\beta$ the asymptotic expression for $\psi_l(r)$ at large distances is

$$|\psi_l\rangle_{r\to\infty} = \frac{e^{ik_0 r}\, 2\lambda_0 r_a \psi_v^{(h_0)}(0)}{r(1-\lambda^2\langle g|G_h|g\rangle\langle g|G_l|g\rangle} = \frac{e^{ik_0 r} f_{hl}}{r}. \qquad (4)$$

It is seen therefore that the quantity $\psi_v^{(h0)}(0)$, which stems from the estimate (3), enters in f_{hl} as a factor, and leads, as will be shown below, to an increase of the amplitude f_{hl} (λ_0 is a dimensionless constant: $\lambda=\lambda_0/4\pi r_a^3\sqrt{m\mu}$, $k_0^2=\mu[E+2(m-\mu)]$).

We note that the exact solution of the problem (2) in explicit form corresponds to the following sequence of summing the perturbation-theory diagrams: the entire perturbation-theory series with respect to the potential V_h is summed in each order in the constant λ (since we have used the exact Green's function G_h throughout). As noted in[3], this sequence of summing the diagrams leads to an answer that does not coincide with the so called "exact solution" of the problem of bound-states in an optical poten-

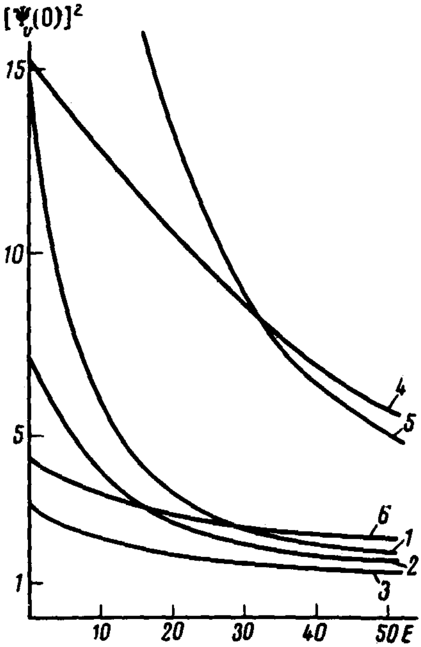

FIG. 1. Dependence of the enhancement coefficient $|\psi_v(0)|^2$ for a square well on the energy of the colliding particles (in MeV); well radius 1.2 F. The different plots correspond to different values of the potential depths: $U_1=70$ meV; $U_2=90$ meV; $U_3=120$ meV; $U_4=520$ meV; $U_5=550$ meV; $U_6=700$ meV. The enhancement coefficients for U_4 and U_5 have been decreased by a factor of two.

tial, a solution that regards annihilation as the imaginary part of the potential.[4] Owing to the incorrect sequence of summation of the diagrams in the optical approach, the enhancement coefficient $|\psi_v(0)|^2$, which follows from (3), does not appears for the annihilation cross section. To describe the large annihilation, it was necessary to introduce in[4] a large radius of the imaginary part of the potential, and this led to vanishing of the bound states in the $N\bar{N}$ system.

In our approach the amplitude f_{hl} turns out to be large also at a small annihilation radius r_a. In fact, it can be shown that if there are no narrow resonances in the heavy-particle scattering amplitude without allowance for the coupling of the channels, then the exact Green's function G_h in the denominator of expression (2) for $|\psi_l\rangle$ can be replaced by the free Green's function G_{h0}. The amplitude f_{hl} then takes the form

$$f_{hl} = \frac{2\lambda_o r_a}{1-\lambda_o^2\,\beta^4(\beta-ip_o)^{-2}\,(\beta-ik_o)^{-2}}\,\psi_v^{(ho)}(0), \tag{5}$$

where $p_0^2=mE$. It follows from (5) that at $|\lambda_0|\gtrsim 1$ the amplitude f_{hl} is of the order of $r_a\psi_v^{(h0)}(0)$, and we arrive at Shapiro's formula for the annihilation cross section

$$(\sigma v)_{\rm an} = (\bar{\sigma} v)_{\rm an}\,|\,\psi_v(0)|^2, \tag{6}$$

where $(\bar{\sigma}v)_{\rm an}\sim 4\pi r_a^2\,k_0/m\sim 2.5$ mb.[2)]

We now investigate the enhancement coefficient $|\psi_v(0)|^2$. In the general case[5] it is expressed in terms of the Jost function $f(-p)$:$|\psi_v(0)|^2=1/|f(-p)|^2$. At low energies, when the scattering amplitude in a potential V_h that is finite at zero is determined by the nearest pole with a binding energy ϵ_b we can present for $|\psi_v(0)|^2$ the simple formula

$$|\psi_v(0)|^2 = \frac{U_0 + E}{\epsilon_b + E} \frac{1}{|\phi(v)|^2}, \tag{7}$$

in which U_0 is the depth of the well. The function $\phi(v)$ depends on the details of the potential, and for order-of-magnitude estimates it can be replaced by unity. Thus, at $U_0/\epsilon_b \gg 1$ the enhancement coefficient $|\psi_v(0)|^2$ can be large. To illustrate the foregoing, Fig. 1 shows the enhancement coefficients $|\psi_v(0)|^2$ for a square well. Realistic models of the $N\bar{N}$ interaction correspond to well depths U_0 such that the well contains one or two levels. It is seen that on the average, with increasing depth of the well the enhancement coefficient increases and becomes very large at those values of U_0 at which the next level occurs. The quantity U_0 can be estimated from the shifts of the Coulomb levels of the $p\bar{p}$ atom.[6] An upward shift of the 1S level by 2 keV corresponds to an enhancement coefficient $|\psi_v(0)|^2 = 7.3 (U_0 \approx 90$ meV$)$, in which case $(\sigma v)_{an} = 18$ mb. In the case of the same shift of the Coulomb level coming from the second level in the U_0 well, the enhancement coefficient and correspondingly the cross section turn out to be larger by one order of magnitude. Thus, large annihilation cross sections are obtained in our model at small annihilation widths (see[1]).

We note in conclusion that, as follows from Fig. 1, the enhancement coefficient $|\psi_v(0)|^2$ decreases with increasing energy. This should lead to a deviation of the annihilation cross section at low energies from the $1/v$ law: the quantity $(\sigma v)_{an}$ should increase with decreasing v. The character of the growth is determined by the quantity $\epsilon_b + E$. At $U_0 \gg E \gg \epsilon_b$ the value of $(\sigma v)_{an}$ increases like $1/v^2$, and the smaller ϵ_b the steeper the growth of $(\sigma v)_{an}$ as $v \to 0$.

The authors thank I.S. Shapiro and V.E. Markushin for discussions and valuable remarks.

[1)]This estimate is valid at potentials V_h that are not too deep, such that $mV_h < \beta^2$.

[2)]This figure is obtained at $\mu = 780$ meV $= m_\omega$. At $\lambda_0^2 \approx 1$ the amplitude (4) contains an enhancement of the type β/k_0 in addition to the estimate $f_{hl} \approx r_a \psi_v(0)$. This enhancement is the result of the pole of $\lambda_0^2 = 1$ on the threshold of the light particles as a result of the coupling of the channels, and we disregard this enhancement in the estimates of f_{hl}, since it appears only for the special choice of the constant $\lambda_0^2 \approx 1$.

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