Peak effect in the dependence of the critical current of a superconductor on the magnetic field

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Near the critical field in a type-II superconductor, the magnetic force lines become detached from flexural deformations of the Abrikosov lattice. This effect leads to additional softening of the lattice and to a sharp increase of the critical pinning current.

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A narrow tall maximum was observed in many experiments in the plot of the critical pinning current at magnetic fields close to H_{c2}.[1–3] As H_{c2} is approached, the force of interaction of the individual pinning center with the vortex lattice decreases regardless of the nature of the pinning center. However, the critical current is determined by the collective action of a large number of randomly distributed inhomogeneities. In an absolutely rigid lattice, the average force exerted on the vortices by a large number of pinning centers is zero. Therefore the critical current, which is proprotional to this force, increases with decreasing rigidity of the lattice. As H_{c2} is approached, the shear modulus C_{66} decreases rapidly. As shown in our previous paper,[4] this drop compensates for the decrease of the force exerted by the individual pinning center on the vortex lattice, and produces a plateau on the plot of the pinning force against the magnetic field.

As will be shown below, near H_{c2}, particularly in superconductors with large values of the Ginzburg-Landau parameter κ, an additional softening of the lattice takes place. This softening is caused by the fact that at short-wave flexural deformations the magnetic force lines become detached from the lattice and are bent slightly. The final result depends little on the nature of the pinning centers and we shall assume, for the sake of argument, that the constants of the effective interaction between the electrons is a random function of the coordinates.

$$g^{-1}(\mathbf{r}) = \langle g^{-1}(\mathbf{r}) \rangle + g_1(\mathbf{r}); \qquad \langle g_1(\mathbf{r}) g_1(\mathbf{r}') \rangle = \phi(\mathbf{r} - \mathbf{r}').$$

The free energy takes in the case the form

$$F = F_0 + \nu \int g_1(\mathbf{r}) |\Delta|^2 d^3 r, \qquad (1)$$

here F_0 is the Ginzburg-Landau free energy in a homogeneous superconductor, and $\nu = mp/2\pi^2$ is the state density on the Fermi surface. It is assumed below that the inhomogeneities are weak enough and that short-range order exists in the system. The order parameter Δ and the vector potential A can be chosen in the form

$$\Delta(\mathbf{r}) = \Delta_0(\mathbf{r} - \mathbf{u}) \exp(2ie\mathbf{u}A_0)(1 + s + i\chi); \qquad \mathbf{A}(\mathbf{r}) = \mathbf{A}_0(\mathbf{r}) + \mathbf{A}_1(\mathbf{r}), \quad (2)$$

where $\mathbf{A}_0$ and Δ_0 represent the solution of the Ginzburg-Landau equations in the homogeneous case. In fields close to H_{c2} the quantities $\mathbf{u}$, s, χ, and $\mathbf{A}_1$ vary little over distances on the order of the lattice period. Substituting expression (2) in formula (1), we obtain an expression for the change produced in the free energy by the presence of deformations and inhomogeneities

$$\delta F = \int d^3\mathbf{r} \left\{ \frac{1}{8\pi} (\operatorname{rot} \mathbf{A}_1)^2 + \langle \mathbf{j} \rangle \mathbf{A}_1 + \frac{k_h^2}{8\pi} \left[\left(\mathbf{A}_1 + [\mathbf{B} \times \mathbf{u}] - \frac{1}{2e} \frac{\partial \chi}{\partial \mathbf{r}} \right)^2 \right.\right.$$

$$\left. + \frac{1}{4e^2} \left(k_\psi^2 s^2 + \left(\frac{\partial s}{\partial \mathbf{r}} \right)^2 \right) - \frac{1}{e} B s \operatorname{div} \mathbf{u} \right] + \frac{C_{66}}{2} \frac{\partial \mathbf{u}^\beta}{\partial \mathbf{r}_\alpha} \frac{\partial \mathbf{u}^\beta}{\partial \mathbf{r}_\alpha}$$

$$\left. + \nu g_1(\mathbf{r}) (1 + 2s) |\Delta_0(\mathbf{r} - \mathbf{u})|^2 \right\}, \qquad (3)$$

where

$$C_{66} = 0.24 \frac{(H_{c2} - B)^2}{4\pi} \frac{2\kappa^2 - 1}{((2\kappa^2 - 1)\beta + 1)^2};$$

$$k_h^2 = \frac{2e^2 p^2 v \tau_{tr}}{3T} \langle |\Delta|^2 \rangle \left[1 - \frac{8T\tau_{tr}}{\pi} \left(\psi\left(\frac{1}{2} + \frac{1}{4\pi T \tau_{tr}} \right) - \psi(1/2) \right) \right] \qquad (4)$$

$k_\psi^2 = k_h^2(2\kappa^2\beta - \beta + 1)$, is the induction, and the coefficients α and β run through the values 1 and 2. Minimizing the free energy (3) with respect to $\mathbf{A}_1$, χ, s, and $\mathbf{u}$ we obtain a system of equations for these quantities. After eliminating $\mathbf{A}$, χ, and s, this system takes the form

$$C_{66}K_\perp^2\mathbf{u} + \frac{B^2k_h^2}{4\pi}\left\{\frac{K_z^2\,\mathbf{u}}{\mathbf{K}^2 + k_h^2} + \mathbf{K}_\perp(\mathbf{K}\mathbf{u})\left(\frac{1}{\mathbf{K}^2 + k_h^2} - \frac{1}{\mathbf{K}^2 + k_\phi^2}\right)\right\}$$

$$= \nu\int d^3r\exp(-i\mathbf{K}\mathbf{r})g_1(\mathbf{r})\left(\frac{\partial}{\partial\mathbf{r}} + \frac{4ieB}{\mathbf{K}^2 + k_\psi^2}\mathbf{K}_\perp\right)|\Delta_0(\mathbf{r} - \mathbf{u})|^2$$

$$+ (2\pi)^3\delta(\mathbf{K})[<\mathbf{j}>\times\mathbf{B}]. \tag{5}$$

At $K \ll k_h$, Eq. (5) goes over into the equation obtained in[5,4]. The dependence of the elastic moduli on $\mathbf{K}$ coincides with the results of Brandt.[6]

Near H_{c2}, the modulus C_{66} is small and the transverse component of $\mathbf{u}$ turns out to be large in comparison with the longitudinal component. From (5) we obtained the correlation function of the displacements over distances much larger than the lattice period

$$<(\mathbf{u}(\mathbf{r}) - \mathbf{u}(0))^2> = \int\frac{d^3K}{(2\pi)^3}W(\mathbf{k})\left(1 - \cos\mathbf{K}\mathbf{r}\right)\left[C_{66}\mathbf{K}_\perp^2 + \frac{B^2k_h^2K_z^2}{4\pi(\mathbf{K}^2 + k_h^2)}\right]^{-2},$$

$$W(\mathbf{K}) = \nu^2\int d^3r_1\exp(-i\mathbf{K}\mathbf{r}_1)\phi(\mathbf{r}_1)\left\langle\frac{\partial|\Delta(\mathbf{r})|^2}{\partial\mathbf{r}}\,\frac{\partial|\Delta(\mathbf{r} + \mathbf{r}_1)|^2}{\partial\mathbf{r}}\right\rangle. \tag{6}$$

Calculating the integrals in formula (5) with logarithmic accuracy, we obtain

$$<(\mathbf{u}(\mathbf{r}) - \mathbf{u}(0))^2> = \frac{W(0)}{4\pi^{1/2}}\,\frac{1}{BC_{66}^{3/2}}\left\{\left(\rho^2 + \frac{4\pi C_{66}}{B^2}z^2\right)^{1/2}\right.$$

$$\left. + \frac{1}{2k_h}\ln\left(\frac{\rho^2}{\zeta^2} + \frac{z^2}{\zeta^4}\,\frac{4\pi C_{66}}{B^2k_h^2}\right)\right\}. \tag{7}$$

At large distances, the principal role is played by the first term in (7); this term was obtained in[5]. However, the distances over which order is lost are significant.

The dimension of the region in which short-range order exists is determined from the condition that the displacements become of the order of the lattice period

$$a^2 = \frac{2\pi}{eB\,3^{1/2}} \sim \langle (\mathbf{u}(\mathbf{r}_c) - \mathbf{u}(0))^2 \rangle . \tag{8}$$

To find the critical current, apart from a numerical factor, we use the following approximation. We assume that the entire volume of the superconductor breaks up into regions whose dimensions are determined by Eq. (8). Inside these regions there exists a regular lattice, and the regions are not correlated with one another. The average force exerted on the lattice by the inhomogeneities is determined by averaging Eq. (5) over such a region. The critical current is reached if these forces, which act in different regions, have the same direction. This is possible, since the regions are not correlated. Carrying out this averaging, we obtain

$$j_c^2 B^2 \sim W(0)/V_c , \tag{9}$$

where V_c is the volume of the region determined by formula (8). In fields that are not too close to H_{c2} and in the case of sufficiently weak pinning, V_c is large and is determined by the first term of (7). In this case formula (9) goes over into the corresponding expression of[4]. In this region both $W(0)$ and V_c are proportional to $(H_{c2}-B)^2$ and a plateau exists in the dependence of the critical current on the field. As H_{c2} is approached, the dimension of the region V_c decreases and the second term in formula (7) becomes substantial. The magnetic field at which the growth of the critical current sets in is expressed in terms of the value of the current $j_{\rm pl}$ on the plateau by means of the formula

$$H_{c2} - B = \left[\frac{2\pi}{e^{1/2}}\, j_{\rm pl}\, B^{3/2} \kappa^4 \ln^2\left(\frac{1}{k_h\, \zeta(T)}\right)\right]^{1/3} \tag{10}$$

With further approach to H_{c2}, the dimension of the region V_c decreases exponentially, and the current increases exponentially

$$j_c \sim \exp\left\{- b\, \frac{(H_{c2} - B)^{3/2} e^{1/4}}{\kappa^2 B^{3/4} j_{\rm pl}^{1/2}}\right\} , \tag{11}$$

where b is a number of the order of unity. The region of the exponential growth of the critical current breaks up into two: in the first, only the transverse dimension ρ_c decreases exponentially, and when H_{c2} is approached z_c likewise begins to decrease exponentially. The coefficient b increases in this case by a factor 1.5.

The exponential growth of the current is stopped when the transverse dimension ρ_c of the region becomes of the order of $\zeta(T)$. At this point j_c is of the order of

$$j_c \sim \frac{1}{10} B^{1/2} \kappa^{2/3} e^{1/6} j_{\rm pl}^{2/3}$$

With further increase of the field, the current apparently decreases. In this region there is not even short-range order in the vortex lattice.

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