Contribution to the theory of thermal modulation instability

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We investigate the nonlinear stage of the development of wave modulation instability produced in a magnetized plasma when the latter is heated. The analysis is confined to a one-dimensional model with modulation of the density transverse to the magnetic field. Numerical solutions and theoretical estimates are obtained for the parameters of the resultant structure.

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It is shown in[1] that the threshold of parametric instability is substantially lowered if the electrons are magnetized. In this case, in contrast to ordinary modulation instability, the modulation of the density is the result of inhomogeneous heating of the plasma, and not by the striction forces. This thermal modulation instability leads to the formation of plasma inhomogeneities that are elongated in the direction of the magnetic field, since the temperature and the density become equalized much faster along the field than across the field. This makes it possible to consider in first-order approximation the development of the modulation of the density in a one-dimensional case, perpendicular to the magnetic field.

The electrons are assumed magnetized $\omega_e \equiv eH/m_e c \gg \nu$, where ν is the electron-atom collision frequency. The high-frequency electric field will be described by an equation obtained by averaging over the high frequency $\omega = \omega_p + \omega_e^2/2\omega_p$, in analogy with[2]:

$$i\frac{\partial E}{\partial t} + \frac{\nu}{2}E + \frac{3}{2}\omega_p l_d^2 \frac{\partial^2 E}{\partial x^2} = \frac{\omega_p}{2}\frac{\delta n}{n}E + (\Delta + i\nu/2)E_0 e^{-i\Delta t}. \quad (1)$$

Here E is the amplitude of the electric field, $\omega_p = \sqrt{\dfrac{4\pi e^2 n}{m_e}}$, $l_d = \dfrac{1}{\omega_p}\sqrt{\dfrac{T_e}{m_e}}$, $\Delta = \omega_0 - \omega$, E_0 and ω_0 are the amplitude and frequency of the pump field.

Slow quasineutral oscillations of the density and of the temperature ($\partial/\partial t \ll \nu$) are described by the equations

$$\frac{\partial}{\partial t}\frac{\delta n}{n} = \frac{\nu T_e}{m_e \omega_e^2}\frac{\partial^2}{\partial x^2}\left(\frac{\delta n}{n} + \frac{\delta T_e}{T}\right), \quad (2)$$

$$\frac{\partial}{\partial t}\frac{\delta T_e}{T} = \frac{5}{2}\frac{\nu T_e}{m_e \omega_e^2}\frac{\partial^2 \delta T_e}{\partial x^2 T} - \delta_\epsilon \nu \frac{\delta T_e}{T} + \frac{2}{3}\nu \frac{|E|^2}{8\pi n T_e}. \quad (3)$$

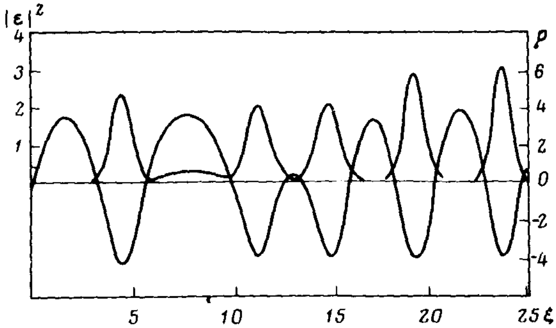

FIG. 1. Spatial distribution of the plasma density $(\delta n/n)=(4\nu/\omega p)\rho$ and of the field $|E|^2=|\epsilon|^2 24\pi n T_e$ at $\tau\equiv 2\nu t=40$. Here $(\omega_e^2/\omega_p^2)=15(\nu/\omega_p)$, $x=(\sqrt{3}/2)l_d(\omega_p/\nu)^{1/2}\xi$.

In the derivation of (2) and (3) we assume $T_i \approx T_a = T$ and neglect the thermoelectric power and the heat flux connected with the electron current, which are small in a weakly ionized plasma. It can be shown[1] that the condition for the applicability of Eqs. (2) and (3) ($\partial/\partial t \ll \nu$) was satisfied if $|E|^2 \leqslant 4\pi n T_e \omega_e^2/\omega_p^2$. In this case, far from the threshold, $|E|^2 > E_{th}^2 = 16\pi n T_e \nu^2/3\omega_e^2$, in the wave-number region $(k\rho_e)^4 \lesssim (\omega_p/\omega_e)^2|E|^2/12\pi n\ T_e$, an instability develops with a maximum increment $\gamma \approx \nu\sqrt{|E|^2/4\pi n\ T_e}$; here $\rho_e = (1/\omega_e)\sqrt{T_e/m_e}$.

The last term in (3) contains a small factor, the fraction $\delta_\epsilon \sim (m_e/m_a) \ll 1$ of inelastic collisions, and is substantial only at relatively large modulation scales $l \sim \rho_e(m_a/m_e)$. At the same time, the characteristic dimension of the produced inhomogeneities turns out to be much less than $l_\perp \sim \rho_e\left(\dfrac{\omega_e^2}{\omega_p^2}\dfrac{4\pi n\ T_e}{|E|^2}\right)^{\frac{1}{4}}$ (see below). We confine ourselves therefore to a study of the fine-scale structure, although this analysis is not complete, as can be seen from the fact that if no account is taken of the electron cooling (the term with δ_ϵ) then there are no stationary solutions (1)–(3).

Let us estimate the parameters of the produced regions of decreased density in the caviton. From the conditions of the field localization in the caviton we obtain

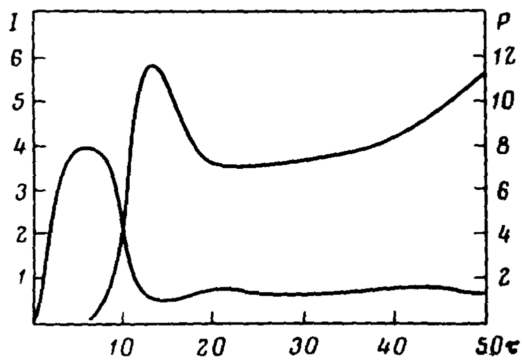

FIG. 2. Time dependence of the degree of dens ty modulation $P=\int\rho^2(d\xi/L)$ and the field ene gy $I=\int|\epsilon|^2(d\xi/L)$.

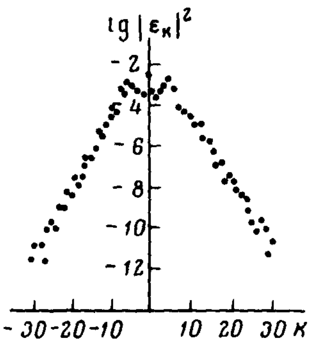

FIG. 3. Spectrum of plasma oscillations at $\tau=40$.

$\delta n_s/n \sim (l_d^2/l_\perp^2)$, where $l_\perp$ is the transverse dimension and δn_s is the decrease of the density in the caviton. The balance between the heat input and heat outflow by thermal conduction yields $\delta T_e/T \sim (l_\perp^2/\rho_e^2) E_s^2/8\pi n T_e$. Inasmuch as at equilibrium we have $\delta T_e/T \approx \delta n_s/n$, we get

$$\frac{\delta n_s}{n} \sim \frac{\omega_e}{\omega_p} \frac{E_s}{\sqrt{8\pi n T_e}}, \qquad l_\perp \sim \rho_e \left(\frac{\omega_e^2}{\omega_p^2} \frac{4\pi n T_e}{E_s^2} \right)^{1/4}, \tag{4}$$

where E_s is the characteristic field in the caviton.

Equations (1)–(3) were solved numerically with periodic boundary conditions, which made it possible to analyze quite fully the small-scale structure of the density modulation. The results of the solution under initial conditions, wherein E are small perturbations and $E_0 = 2\nu\sqrt{24\pi n T_e}$, $\omega_e^2/\omega_p^2 = 15\nu/\omega_p$, are shown in Figs. 1–3. The growth of $P = \int \left(\frac{\delta n}{n}\right)^2 \frac{dx}{L}$ in the quasistationary state is due to the development of ong-wave modulation against the background of a practically stationary small-scale tructure. An analysis of the different variants has shown that the estimates (4) are in atisfactory agreement as a result of a solution in the quasistationary state.

We note that the solution is practically independent of the detuning at $|\Delta| \lesssim \nu$.

The caviton parameters have an interesting dependence on the amplitude of the xternal field. According to the results of the numerical solution we have $E_s^2 \sim E_0$. his relation can be explained by considering the energy balance between the externalurce energy and the rate of plasma heating

$$2\,\mathrm{Im}\{ (\Delta + i\nu/2) E_0 e^{i\Delta t} \int E\,dx \} = \nu \int |E|^2 dx. \tag{5}$$

e use next the estimate

$$\int E dx \approx \frac{2\Delta + i\nu}{i\nu - \omega_p(\delta n/n)_{av}} E_0 e^{-i\Delta t}$$

which follows from Eq. (1). Here $(\delta n/n)_{av}$ is the average depth of the caviton, and $(\delta n/n)_{av} = -\delta n_s/n$. In the case $\omega_p(\delta n_s/n) > \nu > 2|\Delta|$ it follows from (5) that $\left(\omega_p \frac{\delta n_s}{n}\right)^2 E_s^2 l_\perp \sim \nu^2 E_0^2 l_\perp$, and this in conjunction with (4) leads to the relation $E_s^2 \sim (\nu/\omega_p) E_0 \sqrt{4\pi n T_e}$

A numerical solution shows that the average distance λ between the cavitons is proportional to E_0^α, where $\alpha \approx 0.5$; this is in good agreement with the minimal length over which the instability can develop.

The results allow us to conclude that the rate of plasma heating is

$$\frac{d}{dT} < T_e > \approx \nu E_s^2 l_\perp / \lambda \sim E_0^{7/4}$$

and the cross section for scattering of test waves by the resultant structure is $\sigma \sim (\delta n/n)^2 \sim E_0$.

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[1)] In a paper by T. M. Vurinskaya, which will be published in "Fizika Plazmy" [Soviet Journal of Plasma Physics] in 1978.

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