Contribution to the theory of phonon dragging in pure semimetals in a longitudinal magnetic field

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It is indicated that the surface plays a decisive role in the thermoelectric power of phonon dragging of pure semimetals. An experimental situation wherein this effect can be verified is suggested.

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It is well known that in semiconductors and semimetals with low carrier density ıe thermoelectric power at low temperatures is determined by the dragging of the ectrons by the phonons. However, compensated semimetals have a definite distin- ıishing feature compared with doped semiconductors. Its gist is that because of the ıuality of the numbers of electrons and holes the phonon system does not acquire, on e average, a momentum from the carriers. This statement can be qualitatively under- ɔod by starting from Herring's π-approach method and using the Thomson rela- ıns. According to this method, in place of the thermoelectric power it is necessary to lculate the heat flux proportional to the electric field and divide the results by T. hen an electric field is applied, the electrons and holes move in opposite directions, d the flux of the phonons dragged by them is proportional to $n_e - n_h$, where n_e and are the electron and hole densities, respectively. This latter circumstance causes the gging effect to have no effect on the kinetic coefficients if the carrier densities are exactly equal. The corresponding results, if the scattering takes place within the electron-phonon system, can be rigorously corroborated within the framework of the kinetic equation.

However, experimental investigations of the thermoelectric power of very pure bismuth samples reveal in the region of cryogenic temperatures an appreciable increase of the thermoelectric power,[1,2] which exceeds by several orders of magnitude the diffusion emf in this temperature region. The observed temperature dependence seems to point to the appearance of a phonon-phonon dragging mechanism.[3]

It can be shown that the last circumstance is not connected with dissipative processes (U processes, scattering by the boundary) in the phonon system.[4] On the other hand, if we admit the possibility of carrier scattering by any other object, then this leads to decompensation of the system and to a nonzero average phonon drift. However, to explain the observed values of the thermoelectric power, the frequency ν_d corresponding to such a scattering mechanism must be of the order of the carrier-phonon collision frequency ν_{ef}, since the effect is proportional to ν_d/ν_{ef}. Under the experimental conditions,[1,2] the role of this scattering mechanism can be played by carrier scattering from the sample surface, since the mean free paths, say of the electrons, turn out according to the estimates [1] to be of the order of several millimeters, while the mean free path corresponding to the Coulomb interaction of the quasiparticles exceeds the corresponding phonon mean free path by two orders of magnitude.

It is therefore of interest to investigate the thermoelectric power in a magnetic field parallel to the temperature gradient. In this case the magnetic field, by turning the electron and hole trajectories, will effectively decrease in natural fashion the scattering of the carriers by the sample boundaries, and in sufficiently strong fields, whose values will be estimated below, there will be no surface scattering at all. Consequently, with increasing the magnetic field the thermoelectric power should decrease, inasmuch as the phonon dragging makes no contribution in the absence of an additional scattering mechanism.

To investigate the phonon-dragging thermoelectric power of a pure semimetal in a parallel magnetic field, we consider the system of kinetic equations for the carriers and the phonons:

$$\mathbf{v}^{\pm}\nabla_r f_p^{\pm} + e^{\pm}\left\{\mathbf{E} + \frac{1}{c}\left[\mathbf{v}^{\pm}\times\mathbf{H}\right]\right\}\frac{\partial f_p^{\pm}}{\partial \mathbf{p}} = J^N\{f_p^{\pm} G_p\}, \quad (1$$

$$s\,\nabla T\,\frac{\partial G_q}{\partial F} = J^N\{G_q,\ F_q\}, \quad ($$

$$s\nabla T\,\frac{\partial F_q}{\partial T} = J^N\{F_q,\ F_q\} + J^u\{F_q,\ F_q\} + J^N\{F_q,\ G_q\}, \quad ($$

where $v^{\pm}$ and $e^{\pm}$ are the velocities and charges of the carriers, respectively, s is speed of sound, and $f_p^{\pm}$, G_q, and F_q are the distribution functions of the carriers a of the long-wave (electron) and thermal phonons, respectively. The symb

J^N and J^u denote those parts of the collision integrals which correspond to the N and U processes. No account is taken in (2) of the scattering of the phonons by the carriers, since the density of the latter is small. It is permissible to neglect the scattering of the electron phonon by the boundary because at the considered temperatures their mean free path l_f is much smaller than the sample dimension $2b$. This condition, however, does not make the dragging thermoelectric power, which is proportional to the coefficient $(l_f/l)(v_F/s)$ where $(l \sim 2b)$, small, since the Fermi velocity exceeds the sound velocity by three orders of magnitude.

With the indicated approximations taken into account, the dragging effect is accounted for by adding to the energy in the linearized kinetic equation for the carriers an additional term of the type $m^{*\pm} s^2 \frac{\tau_{ph}(p)}{\tau_r^{\pm}}$ where $m^{*\pm}$ is the effective carrier mass, $\tau_{ph}(p)$ is the characteristic relaxation time in the phonon system, and describes the degree of its disequlibrium, while $\tau_r^{\pm}$ is the time of the relaxation of the electrons and holes on the equilibrium phonons.

We seek the solution of (3) in the form

$$f_r^{\pm} = f_r^{\circ} - e^{\pm} \frac{\partial f_r^{\circ}}{\partial \epsilon} \chi_r^{\pm} . \quad (4)$$

In (4), f_r^0 denotes the equilibrium part of the distribution function. For the case of a flat plate bounded by the planes $z = \pm b$, the equation for χ_p, after substituting (4) in (3), takes then the form

$$\frac{\partial \chi_p^{\pm}}{\partial t} + V_z^{\pm} \frac{\partial \chi_p^{\pm}}{\partial z} + \frac{\chi_p^{\pm}}{\tau_r^{\pm}(\epsilon)} = V_x^{\pm} \left[\tilde{E}_x^{\pm}(\epsilon) - \frac{\epsilon - \mu}{Te^{\pm}} \nabla_x T \right],$$

$$\tilde{E}_x^{\pm}(\epsilon) \equiv E_x - \frac{m^* s^2 \overline{\frac{\tau_{ph}(\epsilon)}{\tau_r^{\pm}(\epsilon)}}}{Te^{\pm}} \nabla_x T, \qquad \frac{d\mathbf{p}}{dt} = - \frac{e^{\pm}}{c} [\mathbf{v} \times {}^{\pm}\mathbf{H}] . \quad (5)$$

'he last equation determines the time of motion of the carriers along the trajectory. he boundary conditions for (5), which correspond to diffuse scattering, are

$$\chi^{\pm}_{v_z \gtrless 0}(\pm b) = 0. \quad (6)$$

sing next the standard procedure of solving the kinetic equation [5] we obtain the llowing expression for the contribution made to the average current $\bar{j}_x^{\pm}$ by the dragıg effect

$$\bar{j}_x^{\pm} = \frac{8e^2\sqrt{2m^{*\pm}}}{h^3}\,\frac{r^{\pm}}{b}\int_0^{\infty} d\epsilon\,\frac{\partial f_0}{\partial \epsilon}\,\tau_r^{\pm}(\epsilon)\,\tilde{E}_x^{\pm}(\epsilon)\,\epsilon^{3/2}\int_0^{\pi} d\theta \sin^2\theta\cos^2\theta\int_0^{2\pi} d\phi$$

$$\times \int_{\psi} d\psi \sin(\phi - \psi)\left[\frac{1}{2} - e^{-\frac{\psi r^{\pm}}{l^{\pm}}}\right], \qquad (7)$$

where $r^{\pm}$ is the carrier turning radius in the magnetic field, $l^{\pm}$ is the carrier mean free path, $\phi = \Omega^{\pm} t$ is the phase connected with the motion along the trajectory, and ψ is the angle of the arc along which the electron or hole moves in a plane perpendicular to the direction of the magnetic field from the wall to the point at which its contribution to the current is considered.

Calculating with the aid of (7) the total current from both carrier types and equating it to zero, we obtain the corresponding expressions for the phonon-dragging thermoelectric power in the two limiting cases of weak ($\omega\tau \ll 1$) and strong ($\omega\tau \gg 1$) magnetic fields. In the first limiting case α_{ph} is given by

$$\alpha_{ph} \approx \left(\frac{k}{|e|}\right)\frac{p_F s^2}{kT}\,\frac{\tau_{ph}}{b}\,\frac{(l^- - l^+)}{(l^- + l^+)}\left[1 - \frac{1}{8}\left(\frac{H}{H_0}\right)^2\right] \qquad (8)$$

(H_0 is the field at which $r_0 = l_0$). It follows from (8) that in the weak-field region α_{ph} is independent of the magnetic field and can reach large values at low temperatures.

In the other limit we have correspondingly:

$$\alpha_{ph} \approx \left(\frac{k}{e}\right)\frac{p_F s^2}{kT}\,\tau_{ph}\frac{r_H}{b}\,\frac{1}{(l^+ + l^-)} \qquad r_H = \frac{c\,p_F}{|e|H} \qquad (9)$$

According to (9), in strong magnetic fields α_{ph} decreases in inverse proportion to the magnetic field, since it is proportional to the small parameter r_H/b.

Thus, with increasing magnetic field, the phonon-dragging thermoelectric powei tends to zero. From this point of view, measurements of the thermoelectric power o pure semimetals of the bismuth type in a longitudinal magnetic field would make i possible to reveal the role of surface scattering of the carriers and its contribution t(the anomalously high values of the thermoelectric power of the phonon dragging. Aı estimate for pure bismuth shows that because of the large carrier mean free path ($l \sim$ mm) the corresponding fields in which a substantial decrease of $\alpha_{ph}(H)$ takes plac are of the order of several dozen oersteds.

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