Magnetic-field phase transition in a one-dimensional system of electrons with attraction

G. I. Dzhaparidze and A. A. Nersesyan

Physics Institute, Georgian Academy of Sciences
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Within the framework of an exactly solvable model with a linear spectrum, it is shown that at $T=0$ a magnetic-phase transition from the nonmagnetic into the paramagnetic state, due to the change in the character of the spectrum of the spin excitations, can occur in one-dimensional system electrons with attraction between the particles.

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The influence of the magnetic field on the asymptotic forms of the correlation functions at $h \gg T$ ($h = \mu_B H$, where μ_B is the Bohr magneton) and on the symmetry of the ground state of a one-dimensional system of electrons was considered in [1] in the parquet approximation. This approximation is valid for all values of h if the electron interaction with momentum transfer $\sim 2p_F$ is repulsive, but in the case of attraction, when a gap exists in the system,[2-4] the applicability of this approximation is limited by the condition $h \gg \Delta$. To study the properties of a one-dimensional system in the case of attraction at an arbitrary ratio of h and Δ, we turn to a model with a linear spectrum,[5] which can be solved exactly at $U_{\parallel} = -6\pi v_F/5$, $|U_{\perp}| \ll \pi v_F$, where the constants $U_{\parallel}$ and $U_{\perp}$ describe backward scattering processes without and with spin flip, respectively (v_F is the Fermi velocity).

In this model, using the boson "representation" of fermion fields,[5,6] one changes over to a description in terms of collective excitations of the density (ρ) and of the spin (σ). To study the influence of the magnetic field on the properties of the system it suffices to consider spin degrees of freedom, whose Hamiltonian at $h=0$ is given by

$$\mathcal{H}_\sigma = \frac{2\pi v_F}{L}\sum_{k>0}[\sigma_1(k)\sigma_1(-k) + \sigma_2(-k)\sigma_2(k)] - \frac{U_\parallel}{L}\sum_k \sigma_1(k)\sigma_2(-k)$$

$$+ \frac{U_\perp}{(2\pi a)^2}\int\{\exp[2^{3/2}\pi L^{-1}\sum_k k^{-1}\exp(-a|k|/2 - ikx)(\sigma_1(k)+\sigma_2(k))]$$

$$+ \text{h.c.}\}dx, \qquad (1)$$

where L is the dimension of the system, v_F/a plays the role of the effective width of the band, and $\sigma_n(k) = 2^{-\frac{1}{2}}\sum_{p,s} s\,c^+_{ns}(p+k)\,c_{ns}(p)$ are the operators of the spin density ($n = 1$ or 2 is the number of the field component, $s = \pm 1$ is the spin variable). The $\sigma_n(k)$ algebra[7] makes it possible to define them as density operators of spinless fermions (SF): $\sigma_n(k) = \sum_p a^+_n(p+k)\,a_n(p)$. When using the boson "representation" for the new fermion field,[6] the Hamiltonian $\mathcal{H}_\sigma$ at the point $U_\parallel = -6\pi v_F/5$ becomes equivalent to the single-particle Hamiltonian[5]

$$\mathcal{H}_f = \sum_p pv\,[a^+_1(p)a_1(p) - a^+_2(p)a_2(p)] + \Delta\sum_p [a^+_1(p)a_2(p) + \text{h.c.}]. \qquad (2)$$

with a gap spectrum $E_\pm(p) = \sqrt{p^2v^2+\Delta^2}$, where $v = 4v_F/5$ and $\Delta = |U_\perp|/2\pi a$ (the momenta of the particles are reckoned from $\pm p_F (n = 12)$).

Allowance for the interaction of the electrons with the magnetic field adds to (1) a term $\sqrt{2}h[\sigma_1(0)+\sigma_2(0)]$, which is in turn equivalent to adding to (2) the term, $h\sum_{n,p} a^+_n(p)a_n(p)$ that leads to renormalization of the chemical potential of the SF: $\mu(h) = -h$. At all $h < \Delta$, the value of μ is inside the forbidden band and the ground state of the SF is dielectric. μ penetrates into the lower band $E_-(p)$ at $h > \Delta$, and the system becomes metallic.

The "dielectric–metal" transition in the SF system means a phase transition with respect to the magnetic field in the ground state of the initial electron system, the critical field being $h_c = \Delta$. In the region $h < \Delta$ all the properties of the electron system turn out to be at $T = 0$ the same as in the absence of a field: a gap 2Δ remains in the spectrum of the spin excitations, and the magnetization and the magnetic susceptibility are equal to zero. At $h > \Delta$ the long-wave part of this spectrum, corresponding to the momentum interval $|k| < 2k_0$, where $k_0 = v^{-1}\sqrt{h^2-\Delta^2}$ becomes gapless. In this case a magnetic moment proportional to the number of free states in the lower band of the SF spectrum appears. At $T = 0$ we obtain

$$M = \mu_B\int_{-k_0}^{k_0}\frac{dp}{2\pi} = \frac{\mu_B}{\pi v}\sqrt{h^2-\Delta^2}. \qquad (3)$$

The susceptibility is equal to

$$\chi = \mu_B \frac{\partial M}{\partial h} = \frac{\mu_B^2}{\pi v} \frac{h}{\sqrt{h^2 - \Delta^2}} \quad (4)$$

and diverges as $h \to \Delta + 0$. In strong fields ($h \gg \Delta$) the behavior of M and χ corresponds to the usual Pauli paramagnetism. In this limit, the σ excitations are described only by the Tomonaga-Luttinger model with a line spectrum, which is equivalent to retention of the first two terms of (1).

At finite temperatures, the transition at $h = \Delta$ becomes smeared out, the thermodynamic quantities have no singularities, but they exhibit an anomalous behavior in the region $|h - \Delta| \ll T \ll \Delta$: $M \sim \sqrt{T\Delta}$, $\chi \sim \sqrt{\Delta/T}$ (M and χ are exponentially small at $T \ll \Delta - h$ ($h < \Delta$). We note that in this region the temperature dependence of the specific heat of the system should deviate noticably from linearity, since its spin part $c_\sigma \sim \sqrt{T\Delta}$ exceeds the contribution of the density excitations $c_\rho \sim T$.

At $h = \Delta$, a change should take place in the law governing the decrease of the correlation of the three quantities whose excitation in the absence of a magnetic field entails a loss of threshold energy 2Δ. This pertains primarily to the magnetization correlations, which reduce to correlations of the density fluctuations in the SF system. At $h < \Delta$ we have the exponential law

$$\langle \Delta M(x) \Delta M(0) \rangle = -(\mu_B^2 \Delta / 2\pi |x| \exp(-2\Delta |x| / v), \quad (5)$$

and at $h > \Delta$ the power law

$$\langle \Delta M(x) \Delta M(0) \rangle = -\frac{\mu_B^2}{2\pi^2 x^2} \sin^2 k_o x \quad \text{at} \quad h - \Delta \ll \Delta \text{ and } |x| \gg v/\Delta, \quad (6)$$

$$\langle \Delta M(x) \Delta M(0) \rangle = -\frac{\mu_B^2}{2\pi^2 x^2} \quad \text{at} \quad h \gg \Delta. \quad (7)$$

The situation is analogous with respect to triplet superconducting (TS) and antiferromagnetic (SDW) correlations. At $U_{\parallel} = -6\pi v_F/5$ the contribution of the σ degrees of freedom to the corresponding correlators $K_{TS}(x)$ and $K_{SDW}(x)$ are determined by the mean value[8] $K_\sigma^-(x) = \langle \psi_1^+(x) \psi_2^+(x) \psi_2(0) \psi_1(0) \rangle$, which describes the pair correlations in the SF system. At $h < \Delta$ the presence of the gap in the spectrum of the single-fermion excitations of the Hamiltonian (2) leads to the exponential relation,[8] $K_\sigma^-(x) \sim x^{-2} \exp(-2|x|\Delta/v)$, which gives way at $h > \Delta$ to the power law:

$$K_\sigma^-(x) \sim (v^2/\Delta^2 x^4)(k_o^2 x^2 - \sin^2 k_o x) \quad \text{at} \quad h - \Delta \ll \Delta \text{ and } |x| \gg v/\Delta, \quad (8)$$

$$K_\sigma^-(x) \sim x^{-2} \sin^2(hx/v) \quad \text{at} \quad h \gg \Delta. \quad (9)$$

We note that magnetic-field phase transition described above is not a specific feature of the model chosen by us[5] or of its exact solution for the particular value $U_{\parallel} = -6\pi v_F/5$, but is the consequence of a gap in the spectrum of the spin excitations of the one-dimensional system, a gap that appears in the case of short-range attraction between the particles. This is indicated, in particular, by the results of [9], where numerical solutions of the equations of Lieb and Wu[2] was used to plot the magnetization for the one-dimensional Hubbard model with half-field band in a magnetic field for coupling constants of different values and signs.

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