Excitation of convective cells by Alfven waves

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A nonlinear mechanism is considered for the excitation of convective motions by Alfven waves. The parametric-instability growth rates corresponding to this process are determined and the coupled turbulence of the Alfven wave and of the convective mode is investigated. The question of excitation of convective cells by drift turbulence and of their influence on the anomalous diffusion of the plasma is also considered.

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1. It is known that in the absence of convective instability the corresponding instability mode ($k_{\parallel}=0$, $\mathrm{Re}\,\omega=0$) attenuates slowly as a result of viscosity ($\mathrm{Im}\,\omega=-i\mu k_{\perp}^{2}$, μ is the viscosity coefficient). There exists, however, a mechanism of direct production of convective motions via nonlinear interaction of Alfven waves. In magnetohydrodynamics, the system of equations describing this process is

$$\left(\frac{\partial}{\partial t}-\mu\Delta\right)\Delta\phi=\frac{c}{H}\left[\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right]\Delta_{\perp}\psi; \tag{1}$$

$$\left(\frac{\partial^2}{\partial t^2}-v_A^2\frac{\partial^2}{\partial z^2}\right)\Delta_{\perp}\psi=\frac{c}{H}\left[\left(\frac{\partial^2\psi}{\partial t\partial y}\frac{\partial}{\partial x}-\frac{\partial^2\psi}{\partial t\partial x}\frac{\partial}{\partial y}\right)\Delta\phi+\left(\frac{\partial\phi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\phi}{\partial x}\frac{\partial}{\partial y}\right)\Delta_{\perp}\frac{\partial\psi}{\partial t}\right]. \tag{2}$$

In these equations the transverse components of the electric field in the Alfven wave, as is usual when $\beta=(8\pi nT)/H^2\ll 1$, are represented in the form $E_x=-(\partial\psi/\partial x)$, $E_y=-(\partial\psi/\partial y)$, where $\phi(x,y)$ is the electrostatic potential of the convective mode. Closed cells are produced in this mode as a result of convection of the particles in the crossed fields **H** and $\mathbf{E}=-\nabla\phi$, as shown in Fig. 1, and are analogous to a considerable degree to two-dimensional vortices in an incompressible liquid.

The initial equations take into account only the nonlinearity responsible for the interaction of the convective mode with the Alfven waves. The convective cells are produced as a result of the interaction of Alfven waves having identical ω and $k_{\parallel}$, and

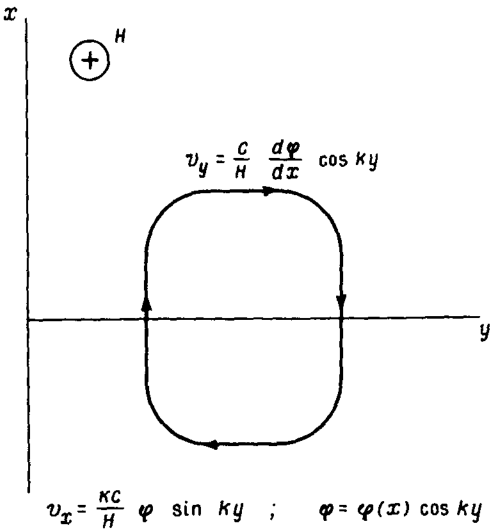

FIG. 1.

correspondingly the superior bar in Eq. (1) denotes averaging over the fast time $\sim 1/\omega$ and over the spatial interval $\sim 1/k_{\parallel}$.

The simplest mechanism of nonlinear production of convective motions is parametric instability of a monochromatic Alfven wave:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}^0 \mathbf{r} - \omega_A t)} + \text{c.c.}, \qquad \omega_A = k_{\parallel}^0 v_A .$$

As is usually the case in parametric instability (see, e.g., [1]) a convective mode is excited with a potential

$$\phi = \phi_{\mathbf{k}} e^{i(\mathbf{k}\mathbf{r}_{\perp} - \Omega t)} + \text{c.c.},$$

as well as two Alfven-wave satellites

$$\mathbf{E} = \left(\mathbf{E}_+ e^{i(k^0 + \mathbf{k})\mathbf{r} - i\Omega t} + \mathbf{E}_- e^{i(\mathbf{k}^0 - \mathbf{k})\mathbf{r} + i\Omega t} \right) e^{-i\omega_A t} + \text{cc.}$$

The instability growth rate $\gamma = \text{Im}\Omega$ can be easily obtained from the initial system of equations:

$$\gamma_A^2 = v_E^2 \, [\mathbf{k}, \mathbf{k}^\circ]_z^2 \frac{1}{|\mathbf{k}_\perp^\circ|^2} \left(1 - \frac{|\mathbf{k}_\perp^\circ|^2}{k^2} \right) \left[\frac{|\mathbf{k}_\perp^\circ|^2}{2} \left(\frac{1}{|\mathbf{k}_\perp^+|^2} + \frac{1}{|\mathbf{k}_\perp^-|^2} \right) - 1 \right], \tag{3}$$

where $v_E = cE_0/H$, $\mathbf{k}_\perp^{\pm} = \mathbf{k}_\perp^0 \pm \mathbf{k}$.

When account is taken of the Alfven-wave frequency dispersion due to the finite

Larmor radius of the ions, $\omega_{\mathbf{k}^0} = k_{\parallel}^0 v_A(1 + k_{\perp}^{02}\rho_i^2/2)$ and $\rho_i^2 = T_e/m_i\omega_{Hi}^2$, the parametric-instability dispersion equation takes the form:

$$\Omega = A\left[\frac{\Delta_-}{\omega_A\Delta_- + 2\Omega}\frac{1}{|\mathbf{k}_\perp^-|^2} - \frac{\Delta_+}{\omega_A\Delta_+ - 2\Omega}\frac{1}{|\mathbf{k}_\perp^+|^2}\right], \tag{4}$$

$$A = v_E^2[\mathbf{k}, \mathbf{k}_o]_z^2\frac{1}{|\mathbf{k}_\perp^o|^2\rho_i^2}\left(1 - \frac{|\mathbf{k}_\perp^o|^2}{k^2}\right), \quad \Delta_\pm = [|\mathbf{k}^\pm|^2 - |\mathbf{k}_\perp^o|^2]\rho_i^2.$$

The magnetohydrodynamic approximation $\rho_i \to 0$ is applicable to our problem only at sufficiently large pump-wave amplitudes $(v_E/v_A) > k^2\rho_i^2$, when the dispersion equation yields the growth rate (3) of the parametric instability. In the opposite limiting case $(v_E/v_A) < k^2\rho_i^2$, instability takes place only for the long-wave satellites in a sufficiently narrow mismatch interval $\Delta_- \sim (v_E^2/v_A^2)(1/k^2\rho_i^2)$ with a maximum value of the growth rate:

$$\gamma_A^2 = \frac{A^2}{\omega_A^2 k^2}\left[\frac{1}{|\mathbf{k}_\perp^-|^2} - \frac{1}{|\mathbf{k}_\perp^+|^2}\right]. \tag{5}$$

2. The instability considered by us produces an additional channel for the damping of Alfven waves via transfer of their energy to convective cells. Such a channel may be quite significant for the solar corona, where one of the most probable mechanisms of plasma heating is connected with the rapid dissipation of the Alfven waves that emerge from the deeper regions of the sun.[2] The situation here is somewhat different from the above-described instability of a monochromatic wave. As a result of the nonlinear wave interaction, an ensemble of convective cells of various scales is produced in the Alfven turbulence. The cell production is described by the equation

$$\frac{d^2\phi_{\mathbf{k}}}{dt^2} + \mu k^2\frac{d\phi_{\mathbf{k}}}{dt} + \frac{c^2}{2H^2}\frac{\phi_{\mathbf{k}}}{k^2}\int d\mathbf{g}\frac{[\mathbf{k}, \mathbf{g}]_z^2}{|\mathbf{k} + \mathbf{g}_\perp|^2}\Big[(\mathbf{g}_\perp^2 - \mathbf{k}^2)|E_{\mathbf{g}}|^2 - ((\mathbf{k} + \mathbf{g}_\perp)^2 - \mathbf{k}^2)|E_{\mathbf{k}+\mathbf{g}}|^2\Big] = 0, \tag{6}$$

which was derived under the assumption that the Alfven waves have random phases, and replaces formula (3) for the growth rate of the parametric instability of a monochromatic wave.

The subsequent evolution of the convective mode is determined by two factors: nonlinear spectral energy transfer with a characteristic growth rate

$$\gamma_{NL} \sim \frac{c}{H} k^2 [k | \phi_{\mathbf{k}} |^2]^{1/2} \qquad (7)$$

and damping due to viscosity, which is important outside the source region.

The singularities of the convective turbulence are the same as in two-dimensional turbulence of an imcompressible liquid. It was shown in [3,4] that in the latter case the conservation of the velocity curl that is connected with the element of the liquid leads to energy flow into the region of small wave numbers, and this flow manifests itself in a tendency of the vortices to combine and to become larger.

The description of the coupled turbulence of Alfven waves and convective cells is an exceedingly complicated problem. We confine ourselves to a qualitative analysis of the quasistationary state that is established in such a turbulence. In this state, the energy of the Alfven waves excited as a result of any particular linear-instability mechanism goes over, in accordance with (6) into the convective cell, is then "pumped out" of the region of the source of the convective turbulence at a growth rate (7), and is absorbed in final analysis by the plasma as a result of viscosity. From the corresponding balance conditions we can obtain the following estimates for the quasi-stationary level of the Alfven waves and convective motions

$$\int d\mathbf{k} \, | E_{\mathbf{k}} |^2 \sim H^2 \frac{\gamma_L^2}{k_A^2 c^2} , \quad \int d\mathbf{k}_\perp \mathbf{k}_\perp^4 | \phi_{\mathbf{k}} |^2 \sim \frac{\gamma_L}{\mu} \int d\mathbf{k} \, | E_{\mathbf{k}} |^2 \qquad (8)$$

γ_L is the growth rate of the energy pumping into the Alfven turbulence.

3. The formation of large-scale vortex cells can turn out to be quite important in problems of magnetic containment of a plasma, since the anomalous-diffusion theory that has been developed so far must be revised to allow for these vortices. In the containment problem, the convective cells are the result of the nonlinear interaction of the short-wave drift waves with a dispersion law $\omega_D = -k_y D_B (d \ln n_0/dx) \times (1 - k_\perp^2 \rho_i^2)$, where $D_B = \dfrac{cT_e}{eH}$ is the Bohm coefficient. The corresponding growth rate of the parametric instability of the monochromatic drift wave is

$$\gamma_D^2 = \frac{D_B^2}{4} \frac{T_e}{T_i} \frac{\mathbf{k}_0^2 - \mathbf{k}_1^2}{|\mathbf{k}_0 - \mathbf{k}_1|^2} [\mathbf{k}_0 , \mathbf{k}_1]_z^2 \left| \frac{\delta n}{n_0} \right|^2 , \qquad (9)$$

where $\mathbf{k}_0$ and $\mathbf{k}_1$ are the wave vectors of the fundamental and sounding drift waves, which have identical ω and $k_\parallel$, while δn is the density modulation in the fundamental wave. This instability is used to explain the results of a numerical experiment,[5] in which production of convective cells by short-wave ($k\rho_i \sim 1$) drift waves was observed. The instability growth rate [$\gamma_D \sim (1/150)\omega_{pe}$ under the conditions of the experiment] agrees well with the time of the appearance of the cells ($\tau \sim 1000\omega_{pe}^{-1}$).

Thus, in the case of magnetic containment we encounter the problem of strong turbulence of drift waves, due to their nonlinear coupling with slow motions of the plasma in the convective cells. The principal question in this problem is that of the final influence of the convective cell on the plasma transport across the magnetic field. The results depend substantially on the presence of shear of the force lines. The most effective diffusion on the convective cells takes place in the plasma with straight magnetic force lines. In this case, in the stationary state (the energy pumping into the drift turbulence with growth rate γ_L is offset by damping in the convective mode with decrement $\Gamma \sim Dk^2$ due to diffusion), the diffusion coefficient of the plasma turns out to be quite appreciable:

$$D \sim D_B \left[\frac{\gamma_L}{\omega_D} \frac{T_e}{T_i} \right]^{1/2} . \qquad (10)$$

In a plasma with shear, the growth of $k_\parallel$ leads to localization of the convective cells in regions with transverse dimensions $\Delta x \sim (\rho_i/\theta)\sqrt{\beta}$ where θ is the shear and it is assumed that $\beta > (m_e/m_i)$. The onset of effective diffusion in the convective mode is possible only when the regions of localization corresponding to the different modes in k_z overlap, and this calls for satisfaction of the condition $a/R < k\rho_i \sqrt{\beta}$ (a and R are respectively the major and minor radii of the toroidal trap).

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