Energy balance of cosmic rays in multiple scattering in a randomly inhomogeneous magnetic field

L. I. Dorman, M. E. Kats, Yu. I. Fedorov, and B. A. Shakhov

Geophysics Institute, Ukrainian Academy of Sciences
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The equation for the energy density of cosmic rays is used to analyze the variation of their energy following multiple scattering by random magnetic-field inhomogeneities that move with velocity **u**(**r**). It is shown that if the radial gradient of the cosmic rays is positive the particles land in the acceleration regime.

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When cosmic rays (CR) propagate in interplanetary space, energy exchange takes place between the charged particles and the random inhomogeneities of the interplanetary magnetic fields that are frozen into the plasma of the solar wind. The predominant premise in the analysis of the energy dissipation in the CR+solar wind system is the assumption that the cosmic-ray charged particles are adiabatically slowed down as a result of the prevailing probability of overtaking collisions with the radially moving inhomogeneities of the magnetic field. We shall show that these premises are insufficient, since the actual character of the spatial distribution of the particles is ignored; it is therefore necessary to review the notions concerning the character of the propagation of the CR in interplanetary space. Moreover, the sharply inhomogeneous character of the expansion of the solar-wind plasma leads to the presence of a unique CR-acceleration mechanism due to the spatial inhomogeneity of the particle distribution function.

We start with the CR transport equation[1]

$$\frac{\partial N(\mathbf{r},p,t)}{\partial t}-\frac{\partial}{\partial r_\alpha}\kappa_{\alpha\lambda}(\mathbf{r},p)\frac{\partial N(\mathbf{r},p,t)}{\partial r_\lambda}+\mathbf{u}\frac{\partial N(\mathbf{r},p,t)}{\partial \mathbf{r}}-\frac{p}{3}\frac{\partial N(\mathbf{r},p,t)}{\partial p}\,\mathrm{div}\,\mathbf{u}=0, \tag{1}$$

where $N(\mathbf{r},p,t)$ is the density of particles with specified momentum p, $\kappa_{\alpha\lambda}(\mathbf{r},p)$ is the tensor of the diffusion of the particles in space, and $\mathbf{u}(\mathbf{r})$ is the velocity of the solar wind. Summation over the repeated indices is implied in (1) and hereafter.

The equation, corresponding to (1), for the energy density

$$E(\mathbf{r},t)=\int_0^\infty dp\,p^2\epsilon N(\mathbf{r},p,t) \tag{2}$$

f the CR (ε is the total energy of the particle) is of the form

$$\frac{\partial E(\mathbf{r},t)}{\partial t} + \operatorname{div}\mathbf{q}(\mathbf{r},t) = \frac{1}{3}\int_0^\infty dpp^3 v\left(\mathbf{u}\frac{\partial N(\mathbf{r},p,t)}{\partial \mathbf{r}}\right), \tag{3}$$

where

$$\mathbf{q}(\mathbf{r},t) = \int_0^\infty dpp^2\epsilon\mathbf{j}(\mathbf{r},p,t) \tag{4}$$

is the CR energy flux density and

$$j_\alpha(\mathbf{r},p,t) = -\kappa_{\alpha\lambda}(\mathbf{r},p)\frac{\partial N(\mathbf{r},p,t)}{\partial r_\lambda} - u_\alpha(\mathbf{r})\frac{p}{3}\frac{\partial N(\mathbf{r},p,t)}{\partial p} \tag{5}$$

is the CR flux density; $\mathbf{v}=\frac{c^2\mathbf{p}}{\epsilon}$ is the particle velocity and c is the speed of light.

On the other hand, Eq. (1) corresponds to the particle-number conservation law

$$\frac{\partial n(\mathbf{r},t)}{\partial t} + \operatorname{div}\mathbf{I}(\mathbf{r},t) = 0, \tag{6}$$

where

$$n(\mathbf{r},t) = \int_0^\infty dpp^2 N(\mathbf{r},p,t)$$

is the particle density and

$$\mathbf{I}(\mathbf{r},t) = \int_0^\infty dpp^2\mathbf{j}(\mathbf{r},p,t)$$

is the flux density of the CR with all energies.

Equation (3) has the same form as a continuity equation with a source in the right-hand side, and the sign of the source determines the character of the change in the CR energy. As follows from (3), the sign of the term corresponding to the source, in the case of radial outflow of the solar-wind plasma, is determined by the direction of the radial gradient of the CR, and if the radial gradient is positive (as is the case for galactic CR) this term represents the amount of energy acquired by the particles per unit volume and per unit time when they interact with the moving inhomogeneities of the magnetic field. Thus, in this case the total number of particles, in accordance with Eq. (6), is conserved, and the particle energy density increases—a situation typical of the presence of particle acceleration. If the radial gradient of the CR is negative, then the inverse process takes place—the particles give up energy to the inhomogeneities of the magnetic field and are slowed down.

The same conclusion concerning the character of energy exchange between the CR and the moving inhomogeneities of the magnetic field follows from Eq. (1). This equation is of the Fokker-Planck type, and to determine the physical meaning of the kinetic coefficient contained in this equation it must be rewritten in canonical form, i.e., in the form of the condition for the conservation of the number of particles in phase space:

$$\frac{\partial N(\mathbf{r},p,t)}{\partial t} + \operatorname{div} \mathbf{j}(\mathbf{r},p,t) + \operatorname{div}_p \mathbf{J}_p(\mathbf{r},p,t) = 0, \tag{7}$$

where $\mathbf{J}_p(\mathbf{r},p,t)$ is the particle flux density in momentum space, and the subscript p of the operator div_p denotes that in this case it is necessary to take into account only that part of the divergence operator in momentum space, which depends on the modulus of the momentum. Taking (7) into account, we rewrite (1) in the form

$$\frac{\partial N(\mathbf{r},p,t)}{\partial t} + \frac{\partial}{\partial r_a}\left\{-\kappa_{a\lambda}(\mathbf{r}, p)\frac{\partial}{\partial r_\lambda} + D_{ap}\frac{\partial}{\partial p}\right\} N(\mathbf{r},p,t)$$

$$+\frac{1}{p^2}\frac{\partial}{\partial p} p^2 D_{pa}\frac{\partial N(\mathbf{r},p,t)}{\partial r_a} = 0, \tag{8}$$

where

$$D_{ap} = - D_{pa} = -\frac{1}{3} p u_a \tag{9}$$

are the components of the particle-diffusion crossing tensor, and describe the exchange of energy between the CR and the magnetic-field inhomogeneities frozen into the solar-wind plasma. In accordance with the general theory, the quantities D_{ap} and D_{pa} (as well as $\kappa_{a\lambda}$) satisfy the principle of symmetry of the kinetic coefficients. As follows from (8), the particle flux density vector in momentum space is determined by the expression

$$J_p = D_{pa}\frac{\partial N(\mathbf{r},p,t)}{\partial r_a} .$$

It should be noted that in the initial formulation of the problem of CR propagation[2-4] the form of the Fokker-Planck equation was postulated (in contrast to [1], where the first consistent derivation of this equation was obtained directly from the kinetic equation) on the basis of the concept of the systematic energy lost by the particles when they interact with the radially diverging inhomogeneities of the magnetic field. An incorrect expression was used there for the particle flux $\mathbf{j}(\mathbf{r},p,t)$ in space, and the particle flux in momentum space was defined by the expression

$$J_p = \left\langle\frac{\partial p}{\partial t}\right\rangle N(\mathbf{r}, p, t)$$

where the kinetic coefficient $\left\langle\frac{\partial p}{\partial t}\right\rangle$ has the meaning of the change of the particle momentum per unit time; it was calculated by resorting to intuitive considerations based on the assumption that the particles lose energy systematically. Although these erroneous assumptions yielded a perfectly correct transport equation, it is incorrect to regard this equation as canonical[1)] and the interpretation, based on this equation, of the physical phenomena that take place when CR propagate in interplanetary space is incorrect. Moreover, a consistent phenomenological treatment does not require at all the calculation of the kinetic coefficient $\left(\frac{\partial p}{\partial t}\right)$ and, as seen from (8), it is necessary to determine the crossing diffusion coefficient $D_{p\alpha}$, which characterizes the energy exchange between the CR and the magnetic inhomogeneities, due to the spatial inhomogeneity of the distribution function of the particles in accordance with the general conclusion that follows from (3). Thus, the concept of adiabatic slowing down of particles is not global, and the energy exchange in the CR+solar wind system is determined by the concrete form of the particle distribution function. The galactic CR propagating in the solar wind are thus accelerated and acquire energy in the course of scattering by the radially moving magnetic-field inhomogeneities.

We cite in conclusion relations that follow from (8) and determine the change of the momentum (energy) of the particle per unit time:

$$\frac{\partial p}{\partial t} = \frac{p}{3}\left(\mathbf{u}\,\frac{1}{N}\,\frac{\partial N}{\partial \mathbf{r}}\right), \quad \frac{d\epsilon}{dt} = \frac{\alpha\epsilon}{3}\left(\mathbf{u}\,\frac{1}{N}\,\frac{\partial n}{\partial \mathbf{r}}\right),$$

$$\alpha = \frac{\epsilon + 2mc^2}{\epsilon + mc^2}$$

where m is the rest mass of the particle.

It is also seen from these relations that the change of the particle energy is determined by the sign of the radial gradient of the cosmic rays and the particle energy increases in the case of a positive radial gradient. A feature of the presented relations is that the average change of the particle energy is determined by the value of the relative gradient of the cosmic rays, a parameter that characterizes the collective properties of the considered ensemble of particles.

[1)]In subsequent studies (see, e.g., [5]), a correct expression was used for the particle flux j (see (5)), corresponding to that obtained in [1], but, as before, an incorrect canonical form of the transport equation was postulated.

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