Correlation method of measuring the parameters of coherent-radiation beams

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The correlation of the intensity of coherent radiation scattered by a random cluster of particles has revealed an anomaly that depends strongly on the curvature of the front of the incident wave. It is proposed to use this anomaly, for example, to determine the divergence of a light beam.

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It was shown in[1] that in the far zone the correlation function of the intensity of light singly scattered by a system of particles at scattering angles larger than $\lambda/2\pi l$, where λ is the wavelength of the light and l is the minimal scale of the inhomogeneity in the distribution of the particle coordinates, contains two terms

$$K(\mathbf{R},\ \mathbf{R}') = \mathrm{const}\left\{ \int\limits_V \int\limits_V W_{ik} \exp\left[\frac{2\pi j}{\lambda R_o} (\mathbf{R} - \mathbf{R}')\mathbf{r}_{ik}\right] \frac{d^2 V}{V^2} \right.$$

$$+\int_V\int_V W_{ik}\exp\left[-\frac{2\pi j}{\lambda R_0}(\mathbf{R}+\mathbf{R}')\,\mathbf{r}_{ik}\right]\frac{d^2V}{V^2}\Bigg\},\qquad(1)$$

where W_{ik} is the pair distribution function of the particle corrdinates in the volume, $\mathbf{r}_{ik}=\mathbf{r}_i-\mathbf{r}_k$, and $\mathbf{r}_i$ and $\mathbf{r}_k$ are the coordinates of the particles, V is the volume of the system of particles, R_0 is the radius of the spherical observation surface, the center of which coincides with the center of the volume with the particles, and $\mathbf{R}$ and $\mathbf{R}'$ are vectors drawn from the point of intersection of the axis passing through the medium of the sounding beam of light and the observation surface at two points on this surface.

The initial light beam was assumed homogeneous and with a plane wave front.

It follows from (1) that $K(\mathbf{R},\mathbf{R}')$ has two maxima corresponding to the two terms and equal in magnitude, which are reached at $\mathbf{R}=\mathbf{R}'$ and $\mathbf{R}=-\mathbf{R}'$. The last condition, naturally, can be satisfied on a sphere only approximately in the region of small scattering angles. If this condition is satisfied then, obviously, it is possible to record the correlation at observation-surface points that are opposite relative to the light-beam axis.

If the sounding light wave front deviations from an ideal plane, i.e., if the amplitude of the wave is such that $E_0=(\mathbf{r})\exp[j\phi(\mathbf{r})]$, then the integrals in (1) should contain, respectively, the factors[1]

$$E_0(\mathbf{r}_i)E_0^*(\mathbf{r}_k)E_0(\mathbf{r}_k)E_0^*(\mathbf{r}_i)=I(\mathbf{r}_i)I(\mathbf{r}_k),$$

$$E_0(\mathbf{r}_i)E_0^*(\mathbf{r}_k)E_0(\mathbf{r}_i)E_0^*(\mathbf{r}_k)=I(\mathbf{r}_i)I(\mathbf{r}_k)\exp[2j(\phi_i-\phi_k)],\qquad(2)$$

where I is the intensity of the light.

Let us see how the ratio of the maxima of the correlation function is influenced, for example, by the sphericity of the wave incident on the particles. We put to this end $W_{ik}=1$, i.e., we assume that the particles are uniformly arranged, and that the intensity of the incident light is constant over the beam cross section. Then, obviously,

$$a=\frac{K(\mathbf{R},\mathbf{R})}{K(\mathbf{R},-\mathbf{R})}=\int_V\int_V\exp\{2j[\phi(\mathbf{r}_i)-\phi(\mathbf{r}_k)]\}\frac{d^2V}{V^2}.\qquad(3)$$

We assume now that the scattering volume (the sample with the particles) is of the form of a cylinder of diameter d and height h with a generator parallel to the light-beam axis, and that the distance between the center of the spherical wave surface is H. It follows then from simple geometrical calculations that in the far zone, i.e., at $d \ll H$ and $h \ll H$, and if it is assumed that $\phi=0$ on the system axis, the phase shifts will depend only on the distance r of the particles from the axis:

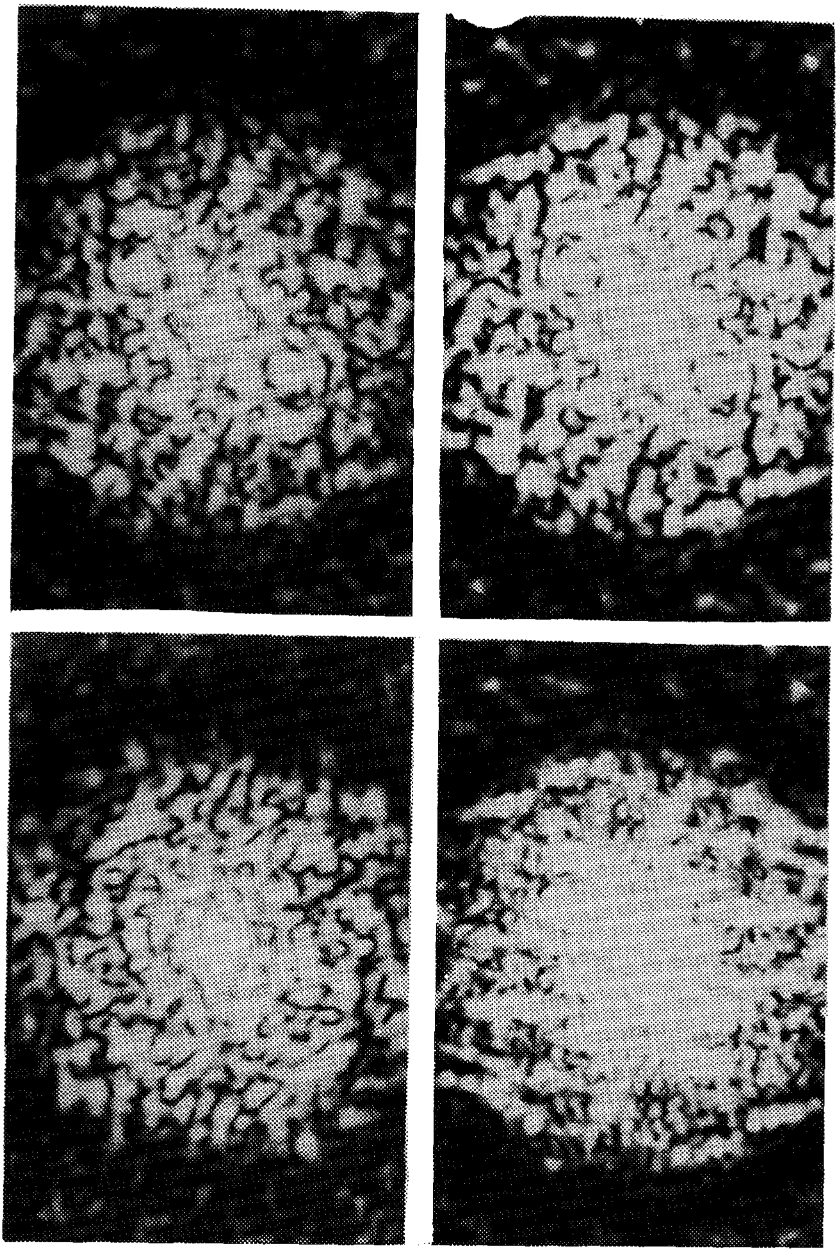

FIG. 1.

$$\phi(\mathbf{r}_i) = \frac{2\pi}{\lambda}\frac{r_i^2}{2H}, \qquad \phi(\mathbf{r}_k) = \frac{2\pi}{\lambda}\frac{r_k^2}{2H}, \tag{4}$$

where r_i and r_k are the distances of the i-th and k-th particles from the light-beam axis.

Substitution of (4) in (3) yields, in cylindrical coordinates,

$$\alpha = \left(\frac{2\pi h}{V}\right)^2 \int_0^{d/2} r \exp\left[j\frac{2\pi r^2}{\lambda H}\right] dr \int_0^{d/2} r \exp\left[-j\frac{2\pi r^2}{\lambda H}\right] dr = \left\{\frac{\sin\left[\frac{\pi}{\lambda H}\left(\frac{d}{2}\right)^2\right]}{\frac{\pi}{\lambda H}\left(\frac{d}{2}\right)^2}\right\}^2 . \tag{5}$$

It follows therefore that only for parallel beams with plane wavefronts, i.e., at $H \to \infty$, we have $\alpha = 1$. In all other cases this is not so. We write down the condition for the first zero of the function (5)

$$\frac{\pi}{\lambda H}\left(\frac{d}{2}\right)^2 = \pi \qquad \text{or} \qquad \gamma = \frac{d}{2H} = 2\frac{\lambda}{d} . \tag{6}$$

This condition means that in order for the "anticorrelation" (α) to be noticeable, it is necessary that the divergence parameter γ be smaller than the quantity $2\lambda/d$. The diffraction limit for γ is equal to $1.22\lambda/d$. Substitution of this value in (5) yields $\alpha \approx 0.24$. Obviously, a correlation of this order can be measured with the aid of an ordinary correlator, and the strong dependence of the divergence makes it possible to use the results of such measurements to verify the coherence of light beams. If the light front is even more plane, the "anticorrelation" waves can be observed even visually.

For example, Fig. 1 shows photographs of the spatial distribution of the intensity of a light beam from a single-mode He-Ne laser scattered in the small-angle region by a cluster of particles. The scattering objects were particles of lycopodium pollen deposited on one side of a glass plate. The wavefront of the initial beam of 4 mm with approximate divergence $2\lambda/d$ was straightened[2] in the focus of a lens of one diopter. The upper left photograph shows the distribution of the intensity in the observation plane at a distance 600 mm from the scattering medium, the latter being located at the focus of the lens. The upper right is a print of two negatives obtained as in the first case, but shifted relative to each other only by rotation through 180° about the center of the illumination. The lower-right photograph shows by way of comparison the same, but the rotation is supplemented by a lateral displacement. The distribution of the intensity in the observation plane in the case when the scattering object was not at the focus but 100 mm away from it along the system axis is shown in the lower left photograph. The "anticorrelation" of the intensity, for the case fixed on the first photograph, is apparently subject to no doubt.

Thus, the correlation of the intensity of coherent light scattered by a cluster of particles, in directions opposite relative to the axis of the sounding beam, has a strong dependence on the curvature of the front of the scattered wave, and this can be used to verify the nature of this front.

[1]B.F. Poltoratskiĭ and K.N. Sachkov, VINITI Deposited paper No. 3908-76, of 10 February 1976.
[2]M. Born and E. Wolf, Principles of Optics, Pergamon, 1971.