Frequency singularities of the dissipation in the mixed state of pure type-II superconductors at low temperatures

N. B. Kopnin

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
(Submitted 16 February 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **27**, No. 7, 417–420 (5 April 1978)

The dissipative part of the conductivity tensor in extremely pure superconductors at low temperatures has a peak at a frequency $\omega \sim \Delta^2/E_F$, due to the presence of bound states in the vortex cores.

PACS numbers: 74.60.Ec

Pure type-II superconductors in the mixed state have a number of interesting features. Thus, the conductivity and the Hall angle in a constant electric field, at low temperatures $T \ll \Delta$, depend significantly on the electron free path time relative to collisions with impurities in the region $\tau\Delta \gg E_F/T_c^2$, i.e., in the region where the superconductor should be regarded as pure according to Anderson's criterion: $\tau\Delta \gg 1$. As shown in[1], in magnetic fields $H \ll H_{c_2}$ a transition to a nondissipative motion of the vortices (dragging of the vortices by the incoming superfluid stream) takes place at $\tau\Delta^2/E_F \gg 1$, whereas at $\tau\Delta^2/E_F \ll 1$ the motion of the vortices retains its dissipative character. These properties of pure superconductors are due to the presence of bound states in the vortex cores.[2] The low-lying bound states $\epsilon_\nu(k)$ are characterized by an azimuthal quantum number ν and by a momentum k along the vortex axis. With respect to the azimuthal number, these levels form an equidistant discrete spectrum

 0021-3640/78/2707-0390$00.60

with distance between levels $\partial\epsilon_\nu(k)/\partial\nu \sim \Delta^2/E_F$. It is the ratio between the path time τ and the distance between the levels which determines the indicated signularities of the vortex motion.

It is quite obvious that the presence of bound states should also influence strongly the absorption of a high-frequency electromagnetic field in the frequency region $\omega \sim \Delta^2/E_F \sim 10^8 - 10^9$ sec^{-1}. These effects should manifest themselves most clearly in the extremely pure case $\tau\Delta^2/E_F \gg 1$. The present paper is devoted to a study of these singularities.

We consider below the case of low temperatures $T \ll \Delta$, and assume for simplicity that the Ginzburg-Landau parameter is $\kappa \gg 1$, and that the magnetic field satisfies the condition $H_{c1} < H \ll H_{c2}$.

The presence of an electric field in the bulk of the superconductor is due to electromagnetic induction produced by the motion of the vortex filaments:

$$\mathbf{E} = -i\omega c^{-1} B[\mathbf{n}_H \times \mathbf{u}_\omega], \qquad (1)$$

where B is the magnetic induction, $\mathbf{n}_H$ is a unit vector in the direction of $\mathbf{B}$ and $\mathbf{u}$ is the displacement of the vortex filaments. The connection between the current and the displacement of the vortices is obtained in accordance with[1] from the relation

$$\frac{\pi}{e}[\mathbf{j}_{tr} \times \mathbf{n}_H] = \nu(0) \int \frac{\partial\epsilon}{4} d^2\mathbf{r}\, \mathrm{Sp}\{\hat{g}^{(a)} \nabla \hat{H}\}, \qquad (2)$$

where

$$\hat{H} = \begin{pmatrix} 0 & -\Delta \\ \Delta^* & 0 \end{pmatrix},$$

and the matrix Green's function $\hat{g}^{(a)}$ at $\tau\Delta^2/E_F \gg 1$ is determined by the equation

$$\hat{g}^{(a)}(\mathbf{r}) = \frac{1}{\pi i \nu(0)} \int d^3\mathbf{r}_1\, \hat{G}^R_\epsilon(\mathbf{r},\mathbf{r}_1) \frac{\omega \mathbf{u}_\omega}{2T} \nabla \hat{H} \mathrm{ch}^{-2}\left(\frac{\epsilon}{2T}\right) \hat{G}^A_{\epsilon-\omega}(\mathbf{r}_1, \mathbf{r}). \qquad (3)$$

The expressions for the functions $\hat{G}^{R(A)}_\epsilon$ were obtained in[1]. The pole part of the functions $\hat{G}^{R(A)}_\epsilon$ constitute an expansion in the eigenfunctions of the bound states. In the summation over ν, the main contribution to (2) is made by the circuiting around the poles corresponding to the bound states. As a result we get

$$j_{tr\,i} = \sigma_{ik}(\omega) E_k, \qquad (4)$$

where the conductivity tensor

$$\sigma_{ik}(\omega) = \sigma^{(0)}(\omega)\delta_{ik} + \delta^{(H)}(\omega) e_{ikl} (\mathbf{n}_H)_l$$

takes the form

$$\sigma^{(O)}(\omega) = -i\,\frac{ce}{B}\int_{-p_F}^{p_F}\frac{dk}{(2\pi)^2}\,q^2\,\frac{\partial\epsilon_\nu(k)}{\partial\nu}\,\frac{\omega+\dfrac{i}{\tau_k}}{\left(\dfrac{\partial\epsilon_\nu}{\partial\nu}\right)^2-\left(\omega+\dfrac{i}{\tau_k}\right)^2}\,, \tag{5}$$

$$\sigma^{(H)}(\omega) = \frac{ce}{B}\int_{-p_F}^{p_F}\frac{dk}{(2\pi)^2}\,q^2\left(\frac{\partial\epsilon_\nu(k)}{\partial\nu}\right)^2\frac{1}{\left(\dfrac{\partial\epsilon_\nu}{\partial\nu}\right)^2-\left(\omega+\dfrac{i}{\tau_k}\right)^2}\,. \tag{6}$$

Here $\epsilon_\nu(k)$ is the energy of the bound states[2]:

$$\epsilon_\nu(k) = \int_0^\infty\frac{\nu|\Delta|}{q\rho}\,e^{-2K}\,d\rho\;\Big/\int_0^\infty e^{-2K}\,d\rho\,,$$

$$\tau_k^{-1} = \frac{1}{2\tau}\int_0^\infty(g^R-g^A)\,e^{-2K}d\rho\;\Big/\int_0^\infty e^{-2K}d\rho\,,$$

$$K = \int_0^\rho\frac{m\,|\Delta|}{q}\,d\rho\,,\qquad q = (p_F^2-K^2)^{1/2}.$$

Expressions (5) and (6) are valid in the limit $\tau\Delta^2/E_F\gg 1$, but describe qualitatively the situation also in the case $\tau\Delta^2/E_F\sim 1$. It is seen from (5) that the real part $\mathrm{Re}\,\sigma^{(O)}(\omega)$, which determines the dissipation, has at $\tau\Delta^2/E_F\sim 1$, generally speaking, a peak at the frequency $\omega\sim\Delta^2/E_F$. The exact expression for $\sigma_{ik}(\omega)$ at $\tau\Delta^2/E_F\sim 1$ can be obtained, by determining $\hat{g}^{(a)}$ from the total kinetic equation. An analytic solution of the kinetic equation is possible, however, under certain simplifying assumptions with respect to the form of the potential $|\Delta(\rho)|$ (see, e.g.,[1]).

For extremely pure superconductors $\tau\Delta^2/E_F\gg 1$ the conductivity depends strongly on the relation between ω and $\partial\epsilon_\nu(k)/\partial\nu$. At $\omega\ll\partial\epsilon_\nu(k)/\partial\nu$, Eq. (4), with (1) taken into account, becomes

$$\frac{\pi}{e}\left[\left(\mathbf{j}_{tr}-Ne\frac{\partial\mathbf{u}}{\partial t}\right)\times\mathbf{n}_H\right] = M\frac{\partial^2\mathbf{u}}{\partial t^2}\,,$$

where the left-hand side contains the Magnus force, and the "mass" of the vortex (per unit length) is

$$M = \int_{-p_F}^{p_F}\frac{dk}{4\pi}\,q^2\left(\frac{\partial\epsilon_\nu(k)}{\partial\nu}\right)^{-1}\sim Nm\,\xi_0^2\,.$$

This connection between the current and the displacement **u** leads to the existence of natural oscillations of the vortex-filament lattice with a quadratic dispersion law.

At $|\omega| < \min(\partial\epsilon_\nu/\partial\nu)$ the real part is $\mathrm{Re}\sigma^{(O)}(\omega) = 0$ and there is no dissipation. On the other hand, if $|\omega| > \min(\partial\epsilon_\nu/\partial\nu)$, a pole singularity appears in the integrand of (5), and its contribution yields a finite real part $\mathrm{Re}\sigma^{(O)}(\omega)$:

$$\mathrm{Re}\sigma^{(O)}(\omega) = \frac{c}{B}\frac{e}{4\pi}\frac{q_0^2}{}\,\frac{\partial\epsilon_\nu(k_0)}{\partial\nu}\Bigg/\left|\frac{\partial^2\epsilon_\nu(k_0)}{\partial\nu\partial k}\right|,$$

where k_0 is determined from the condition $\omega = \partial\epsilon_\nu(k_0)/\partial\nu$. Near the edge of the "absorption band," $\omega = \min(\partial\epsilon_\nu/\partial\nu)$, the second derivative $\partial^2\epsilon_\nu(k)/\partial\nu\partial k$ vanishes, and this leads to a root singularity in the absorption. For the sake of clarity we present an expression for $\mathrm{Re}\sigma^{(O)}(\omega)$ in the model of Ref. 3, according to which $|\Delta(\rho)|$ assumes a constant value $|\Delta_\infty|$ at distance on the order $\xi_1 = \xi_0 T/\Delta$. In this case

$$\partial\epsilon_\nu/\partial\nu = \frac{m|\Delta|^2}{q^2}\ln\frac{|\Delta|}{T}$$

and

$$\mathrm{Re}\,\sigma^{(O)}(\omega) = \frac{3\pi}{4}\frac{Nec}{B}\frac{1}{\tilde{\omega}^{3/2}(\tilde{\omega}-1)^{1/2}}\,\Theta(\tilde{\omega}-1),$$

where

$$\tilde{\omega} = \omega\, 2E_F/|\Delta|^2\ln\left(\frac{|\Delta|}{T}\right).$$

[1]N.B. Kopnin and V.E. Kratsov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 631 (1976) [JETP Lett. **23**, 578 (1976)].
[2]C. Caroli, P.G. de Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964).
[3]L. Kramer and W. Pesch, Z. Phys. **269**, 59 (1974).