Concerning the stability of the critical state in type-II superconductors

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Electromagnetic-field and temperature oscillations preceding the jump of the magnetic flux in type-II superconductors are considered.

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The question of the stability of the critical state in hard and combined superconductors has been repeatedly discussed in the literature. The corresponding criteria for the onset of jumps of the magnetic flux, obtained in an approximation linear in the small perturbations, are well known.[1,2] Much has been left undone in the investigation of the evolution of the resultant instability, and accordingly, the finite flux jumps.[1] This paper is devoted to a determination, in the linear approximation, of the region of existence of bounded oscillating perturbations of the temperature Θ, of the electric field **E**, and of the magnetic field **H**.

The evolution of the perturbations of Θ, **E**, and **H** is described by the heat-conduction equation and by the system of Maxwell's equations

$$\begin{cases} \nu\Theta = \kappa\Delta\Theta + \mathbf{j}_c\mathbf{E} \\ \Delta\mathbf{E} = \dfrac{4\pi}{c^2}\dfrac{\partial\mathbf{j}}{\partial t} \end{cases} \tag{1}$$

where $\mathbf{j}=\mathbf{j}_c+\sigma\mathbf{E}+(\partial\mathbf{j}_c/\partial T)\Theta$, ν, κ, σ are respectively the specific heat, the thermal conductivity, and the electric conductivity (in the resistive state) of the superconductor, and j_c is the density of the critical current (for simplicity we consider Bean's model of the critical state $j_c=j_c(T)$.

We seen the solution of the system (1) in the form $\Theta=\Theta(\mathbf{r})\exp(\Gamma t)$, $\mathbf{E}=\mathbf{E}(\mathbf{r})\exp(\Gamma t)$. To determine the spectrum of the eigenvalues Γ it is necessary to impose thermal and electrodynamic boundary conditions on Eqs. (1).

Consider, for example, a thermally insulated semi-infinite sample in an external magnetic field $H\|z$ parallel to the surface (the yz plane). In this case $E(L)=0(L=cH/4\pi j_c$ is the depth of penetration of the magnetic field), the temperature and the heat flux $\kappa\partial\Theta/\partial x$ are continuous at $x=L$, $\partial H(0)/\partial t=\partial E(0)/\partial x=0$, and $\partial\Theta(0)/\partial x=0$. Since $\partial j_c/\partial H=0$, the system (1) consists of linear equations with constant coefficients. From the condition that it have a solution (the vanishing of the corresponding determinant), we easily obtain an equation for the determination of Γ:

$$(\lambda - K_1^2)K_2 \operatorname{tg} K_2 - (\lambda + K_2^2)K_1 \operatorname{th} K_1 = \sqrt{\lambda}(K_1^2 + K_2^2), \tag{2}$$

where

$$\lambda = \Gamma\frac{L^2\nu}{\kappa}, \quad K_{1\,2}^2 = \pm\frac{\lambda(1+\tau)}{2} + \sqrt{\frac{\lambda^2(1-\tau)^2}{4} + \lambda\beta},$$

$$\beta = \frac{1}{\nu j_c}\left|\frac{dj_c}{dT}\right|\frac{H^2}{4\pi}, \qquad \tau = \frac{4\pi}{c^2}\frac{\kappa\sigma}{\nu}.$$

The stability limit of the region $\operatorname{Im}\lambda=0$, $\operatorname{Re}\lambda(\beta,\tau)>0$, as usual, corresponds to $\beta=\beta_c(\tau)$.[2,3] In most superconductors $\tau\ll 1$, i.e., the heat propagation is much slower than the diffusion of the magnetic field. This condition makes it possible to obtain analytically the relation $\beta_c=\beta_c(\tau)$: $\beta_c=\beta_0(1+2\sqrt{\tau})$, $\beta_0=\pi^2/4, \lambda_c=\lambda(\beta_c)=\beta_0/\sqrt{\tau}\gg 1$.[2] Solving (2) at $\beta\sim\beta_c$ and under the condition $\operatorname{Re}\sqrt{\lambda}\gg 1$, we get

$$\lambda_{1\,2} = \frac{\beta-\beta_0 \pm\sqrt{(\beta-\beta_0)^2-4\beta_0\tau}}{2\tau}, \tag{3}$$

As seen from (3), in the region $\beta_0 < \beta < \beta_c$ or $H_0 < H < H_j$ (here H_0 and H_j are determined from the conditions $\beta(H_0) = \beta$, $\beta(H_j) = \beta_c$) the eigenvalue spectrum turns out to be complex: $\lambda = \lambda_0 + i\lambda_1$, with $\lambda_0 > 0$. We note that at $\beta = \beta_0$ we have $\lambda = i\beta_0/\sqrt{\tau} \doteq \tau\lambda_c$.

Thus, in a narrow interval directly ahead of the jump of the magnetic flux $H_0 < H < H_j$, $(H_j - H_0)/H_j = \sqrt{\tau} \ll 1$, oscillations of the electric field and of the temperature can be observed. We consider the situation in the magnetic-field range $H_0 < H < H_j$ in greater detail. The equation (1) and the ensuing relation (2) are valid if a linear connection exists between the current j and the electric field E. The last condition is satisfied only if $E \geqslant E_0(T)$, where $E_0(T)$ is the boundary of the linear section on the $j = j(E,T)$ curve, so that the values of the electric field in the sample greatly influences the character of the observed effects.

Assume that initially there is no electric field. Then even a large fluctuation will decrease after a time $t_1 \sim L^2\nu/\lambda_1\kappa$ to such an extent that the condition $E < E_0(T)$ is satisfied everywhere in the sample, after which the perturbation will certainly attenuate.[2] Consequently, the growth of fluctuations with an increment $\Gamma = \lambda_0\kappa/\nu L^2$ at the initial instant leads to a change of the magnetic flux only by a finite amount. Thus, one will observe in experiment only bounded magnetic-flux jumps proportional to the amplitude of the initial perturbation.

In the investigation of the stability of the critical state in an external magnetic field that varies at a given rate $\dot{H}$, there is initially present an electric field $E \sim (L/c)\dot{H}$. If $(L/c)\dot{H} > E_0(T)$ or $\dot{H} > cE_0(T)/L$, then oscillations of the electric field with amplitude up to $(L/c)\dot{H}$ can in fact develop in the sample. Let us estimate the number N of oscillations observed at a given value of $\dot{H}$. Since the magnetic field lies in the interval $\Delta H = H_j - H_0 = \sqrt{\tau}\, H_j$ during the time interval $\Delta t = H_j\sqrt{\tau}/\dot{H}$, it follows that N can be estimated as the ratio of Δt to the characteristic period of the oscillations, whence $N \sim \Delta t/\mathrm{Im}\Gamma = H_j\kappa/\dot{H}L^2\nu$; it is seen that $N \gtrsim 1$ at $\dot{H} \lesssim H_j(\kappa/\nu L^2)$. Thus, the oscillations can be observed if the rate of change of the external magnetic field is in the range $cE_0(T)/L < \dot{H} < H_j\kappa/\nu L^2$, or

$$\frac{E_0}{H_j} 4\pi j_c < \dot{H} < \tau\, 4\pi j_c \frac{j_c}{\sigma H_j} .$$

It can be shown that in a semi-infinite sample the oscillations are produced at an arbitrary heat outflow from the boundary. It is likewise easy to investigate the evolution of the perturbations in samples of finite dimensions. In particular if the heat outflow is small in this case, then the period of the oscillations turns out to be relatively large (even at $\tau \ll 1$). A similar problem can be solved also for superconductors with $\tau \gg 1$.

We note in conclusion that potential-difference oscillations observed[4,5] in the investigation of flux jumps against the background of an external maagnetic field that grows with constant velocity correspond apparently to the case $\tau \lesssim 1$ and to poor cooling.

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