Crystallization of Yang-Mills fields in superdense matter

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An exact classical solution, in the form of a standing wave with zero energy, is obtained for the Yang-Mills field in a medium.

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The properties of superdense matter consisting of elementary particles that interact in accordance with unified gauge theories of weak, strong, and electromagnetic interactions have recently become the subject of intensive investigations; see, e.g.,[1–3]. It was assumed in all these studies that the matter is gaseous or liquid, but not crystalline. The only exception is a recent paper[4] dealing with the possibility of vector-field condensation analogous to pion condensation.[5] The reason for so little attention to possible crystallization of superdense matter is that in most theories studied to date a multiple state of superdense matter consisting of ultrarelativistic particles is energy-wise unprofitable in the classical approximation, and crystallization can result only from quantum corrections.

It is demonstrated in the present paper that on going to non-Abelian gauge theories, the possibility of crystallization of matter manifests itself even in the classical approximation, and in this sense the crystallization of superdense matter becomes more readily the rule rather than the exception.

The main idea of the paper will be illustrated with a very simple (albeit nonrealistic) $O(3)$-symmetrical theory of a Yang-Mills field with a Lagrangian

$$L = -\frac{1}{4}(G^a_{\mu\nu})^2 \equiv -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\epsilon^{abc} A^b_\mu A^c_\nu)^2 . \qquad (1)$$

Here A^a_μ is a triplet of massless Yang-Mills fields, and $a=1,2,3$. We introduce also fermions with nonzero average current density J^a_μ, the interaction of which with the field A^a_μ is described in the classical approximation by adding the term $-eJ^a_\mu A^a_\mu$ to (1). Let the current J^a_μ differ from zero only for $a=3$ and $\mu=0$, i.e., assume a nonzero average charge density $J^3_0 \equiv Q$ relative to the neutral vector field A^3_μ.

Generally speaking, in non-Abelian theories the current J^a_μ is not conserved and it is not always possible to realize this situation physically. In a number of cases of physical interest, however, such a situation can be realized; in particular, this can be done in the Weinberg model[6] by producing in space a certain density of a globally conserved lepton charge relative to the neutral vector field Z_μ.[7]

Returning to our model problem, we write the Lagrangian (1) with allowance for the interaction of the field A^a_μ with fermions:

$$L = -\frac{1}{4}(G^a_{\mu\nu})^2 - eQA^3_o\,. \qquad (2)$$

It is easy to verify that the Green's function of the spatial components of the fields A^1 and A^2 has in the Coulomb gauge a pole at $k_0=0$, $|\mathbf{k}|=eA^3_0$, i.e., the excitation spectrum of the system contains states corresponding to a standing wave of Yang-Mills fields A^1 and A^2 with a zero energy and with a wavelength $(eA^3_0)^{-1}$. This circumstance prompts us to seek exact solutions of the Lagrangian equations in the theory (2) in the form of a standing wave. It turns out in fact that at least one such solution exists and has in the Coulomb (as well as in the axial) gauge the following form

$$\begin{aligned} A^1_1 &= A^2_1 = C\sin kZ\,, \\ A^1_2 &= A^2_2 = C\cos kZ\,, \\ A^a_3 &= A^3_1 = A^{1,2}_o = 0\,. \end{aligned} \qquad (3)$$

Here

$$C^2 = \frac{Q}{2eA^3_o}\,, \qquad k = eA^3_o\,. \qquad (4)$$

This solution, which is constant in time and periodic in one of the spatial coordinates, is in fact a one-dimensional Yang-Mills crystal.

We note that the classical solution (3), (4) can correspond to a standing wave of arbitrary nonzero amplitude C. The total system energy

$$H = -e^2(A^3_o)^2C^2 + k^2C^2 + eQA^3_o = eQA^3_o \qquad (5)$$

reduces only to the energy of the external currents, i.e., the self-energy of the crystal is zero in the classical approximation at all values of C.

For this reason, the parameters of the standing wave (3) can ultimately be determined only by taking into account the quantum corrections to the effective Lagrangian (2). A preliminary investigation of this process shows that, when account is taken of single-loop quantum corrections, the energy of the crystal is no longer zero and the minimum energy corrresponds to values

$$A^3_o \sim C \sim \left(\frac{Q}{e}\right)^{1/3}. \qquad (6)$$

For a complete analysis of the results it is necessary, however, to investigate thoroughly the higher orders of perturbation theory. Moreover, one cannot exclude the possibility that, as is customary the case,[8] the absolute minimum of the energy corresponds not to the one-dimensional standing wave (3) but to a three-dimensional lattice: we have obtained only the simplest of the solutions of the nonlinear Yang-Mills equations. In any case, the very existence of exact periodic crystal-type solutions (3) and (4) for Yang-Mills fields with zero energy is a rather unexpected and interesting fact. A more detailed discussion of the questions touched upon in the present paper will be published elsewhere.

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