Spin glass with nonmagnetic defects

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(Submitted 6 March 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **27**, No. 9, 477–481 (5 May 1978)

A theory is constructed for spin glasses with $n_m^{1/3} l \ll 1$, where n_m is the magnetic-impurity concentration and l is the mean free path. The specific heat, the magnetic susceptibility, and the resistivity are obtained at high and low temperatures and in the "transition" region.

PACS numbers: 75.40.Bw

The interaction between localized spins placed in a nonmagnetic metal is expressed by the well known formula (see[1])

$$H_{12} = -(J/n)^2 (\mathbf{S}_1 \mathbf{S}_2) \frac{p_0 m}{4\pi^3} \frac{\cos 2p_0 r}{r^3} e^{-r/l}, \tag{1}$$

where J is the electron–impurity exchange energy; n is the density of the host $\sim$ metal atoms; p_0 is the Fermi momentum; l is the electron mean free path. It is customary to consider the case when the mean free path is large, $l \gg n_m^{-1/3}$, where n_m is the magnetic-impurity concentration. In this case the exponential factor can be neglected and the RKKY interaction sets in[2]. Analysis of this interaction is greatly hindered by two properties: it alternates its sign and it is long-range in character. We consider a case which is in principle realizable if the metal has a sufficiently large concentration of nonmagnetic defects. The interaction then becomes short-range, and each impurity spin interacts only with one nearest neighbor.

At the lowest temperatures the bulk of the spins are frozen, and the thermodynamic characteristics, as well as the temperature dependence of the resistivity, are determined by a few weakly-coupled spins, i.e., those whose nearest neighbor is far or those for which $\cos 2p_0 r$ in (1) is close to zero. Since the spin of the nearest neighbor is frozen, the spin under consideration is under the influence of an effective field H equal to

$$H = V_0 q S \exp(-r/l) r^{-3}, \tag{2}$$

where $q = |\cos 2p_0 r|$ and $V_0 = (J/n)^2 p_0 m/(4\pi^3)$.

It is thus necessary to find the corresponding quantity for the given field H, and then average over the probability distribution of the different values of q and r. For example, for the heat capacity per unit volume we have

$$C = n_m \int 4\pi n_m r^2 \exp\left(-\frac{4\pi}{3} n_m r^3\right) dr \; \frac{2}{\pi} \int_0^1 \frac{dq}{\sqrt{1-q^2}} \left\{ \phi\left(\frac{H}{2T}\right) - \phi\left[\left(S + \frac{1}{2}\right)\frac{H}{T}\right] \right\}, \tag{3}$$

 0021-3640/78/2709-0449$00.60

where $\phi(x)=x^2/\sinh^2 x$. At low temperatures the important role in the integral is played by small q. The resultant integral with respect to r is calculated by the saddle-point method near the "saddle" value $r_1=(4\pi n_m l)^{-1/2}$. This yields

$$C=\frac{\sqrt{\pi}}{3}\frac{T}{V_0(2S+1)}(4\pi n_m l^3)^{-3/4}\exp\left[\frac{2}{3}(4\pi n_m l^3)^{-1/2}\right], \tag{4}$$

$$\chi=\frac{1}{3\pi^{3/2}}\frac{\mu^2}{V_0}(4\pi n_m l^3)^{-3/4}\exp\left[\frac{2}{3}(4\pi n_m l^3)^{-1/2}\right] ln\frac{T_0}{T}. \tag{5}$$

(In the calculation of the magnetic susceptibility χ we took into account the fact that the characteristic field H can be arbitrarily oriented relative to the external field.)

To find the temperature dependence of the resistivity we use the general formula of[3]:

$$\rho=\rho_0+\frac{2m^2}{3e^2n_e^2T}\int\omega_{pp'}\,\mathbf{v}(\mathbf{v}-\mathbf{v}')(1-f_0)f_0'\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{d^3\mathbf{p}'}{(2\pi)^3}, \tag{6}$$

where $n_e=p_0^3/(3\pi^2)$ is the electron density, ρ_0 is the contribution of the potential scattering, $f_0(\xi)$ is the Fermi function, $\xi=\epsilon-\mu$,

$$w_{pp'}=n_m(J/n)^2K(\xi_p-\xi_{p'}),$$

$$K(\omega)=\int_{-\infty}^{\infty}dt\,e^{i\omega t}<\mathbf{S}(0)\,\mathbf{S}(t)>,$$

$\mathbf{S}(t)=\exp(i\mathscr{H}t)\mathbf{S}\exp(-i\mathscr{H}t)$, $\mathscr{H}=-\mathbf{HS}$, and $\langle\cdots\rangle$ stands for a thermodynamic mean value. As a result we get:

$$\rho=\rho_0\left\{1+4\pi^2n_mV_0\tau S^2+\sqrt{2\pi}\,[\dot{I}(S)/S]\,T\tau\,(4\pi n_m l^3)^{-3/4}\exp\left[\frac{2}{3}(4\pi n_m l^3)^{-1/2}\right]\right\} \tag{7}$$

where $\rho_0=n_e e^2\tau/m$, $\dot{I}(1/2)=\pi^{2/4}$, $\dot{I}(1)=(8\pi^2/27)-(\pi/3\sqrt{3})$ and $\dot{I}(S\gg 1)=2S+1/2+0(1/2S)$.

Formulas (4), (5), and (7) are valid at $T\ll T_0$, where T_0 corresponds approximately to the spin-interaction energy at $q\sim 1$ at the "saddle-point" distance $(4\pi n_m l)^{-1/2}$

$$T_0=V_0S^2n_m(4\pi n_m l^3)^{3/4}\exp[-(4\pi n_m l^3)^{-1/2}]. \tag{8}$$

At $T\gtrsim T_0$, small q do not play a special role in the integrals and different temperature dependences are obtained. Calculation with the same general formulas yield

$$C=n_m(4\pi n_m l^3)\ln(2S+1)\ln^2(V_0S^2n_m/T)\exp\left[-\frac{1}{3}(4\pi n_m l^3)\ln^3(V_0S^2n_m/T)\right], \tag{9}$$

$$\chi = n_m \mu^2 S(S+1)\, T^{-1} \exp\left[-\frac{1}{3}(4\pi n_m l^3)\ln^3(V_0 S^2 n_m/T)\right], \tag{10}$$

$$\rho = \rho_0 \{1 + 4\pi n_m V_0 \tau\, [S^2 + S\exp\left[-\frac{1}{3}(4\pi n_m l^3)\ln^3(V_0 S^2 n_m/T)\right]\}. \tag{11}$$

These formulas cease to be valid at $T \gtrsim \Theta$, where the "transition temperature" Θ is equal to

$$\Theta = a^{-1} V_0 S^2 n_m \exp[-a^{1/3}(n_m^{1/3} l)^{-1}], \tag{12}$$

where $a = 3\beta_c/4\pi$ and $\beta_c = 3 \pm 0.1$ (see below). At $T \gtrsim \Theta$ the individual weakly-coupled spins cease to play the preferred role.

If $T \gg \Theta$, then we can use the virial-expansion idea developed in[3]. The first-order increments to the expression for the free spins (the latter may be equal to zero) is determined by the pairs located at the "thermal" distance $r(T)$ defined by the condition

$$V_0 S^2 r^{-3}(T) \exp[-r(T)/l] = T. \tag{13}$$

If this distance exceeds l, i.e., $\Theta \ll T \ll T_1$, where

$$T_1 = V_0 S^2/l^3, \tag{14}$$

then the method of[3] yields the following expressions:

$$C = 4\pi n_m^2 l^3 \ln^2(V_0 S^2/Tl^3)[\ln(2S+1) - \frac{1}{4}\ln(4s+1)], \tag{15}$$

$$\chi = (n_m \mu^2/3T)[S(S+1) - (2\pi/3) n_m l^3 S \ln^3(V_0 S^2/Tl^3)], \tag{16}$$

$$\rho = \rho_0 + \rho_s(1 - n_m l^3 a_s),\quad \rho_s = 4m\pi^2 n_m V_0 S(S+1)/n_e e^2,$$

$$a_s = \pi(\frac{2}{3}S + 1)(S+1)^{-1}\ln^3(V_0 S^2/Tl^3). \tag{17}$$

Formulas of the type of (15) and (16) were obtained in[4] for another system, but also with an exponential interaction.

At $T \gg T_1$ we have $r(T) \ll l$ and we obtain the formulas of[3] for the RKKY interaction.

To study the vicinity of the transition we can use the percolation approach. This is exactly the procedure used by Smith,[5] but he considered the RKKY interaction, for which this approach is not valid. We shall assume that the spins do not interact at distances larger than $r(T)$ (see(13)), but interact strongly at shorter distances and are rigidly coupled. We arrive at the so called problem of spheres, in which the condition for the formation of an infinite cluster is (see[6])

$$\begin{aligned} \beta &= \beta_c , \\ \beta &= \frac{4\pi}{3} r^3(T) n_m, \end{aligned} \tag{18}$$

A numerical calculation yields $\beta_c = 3 \pm 0.1$. From (13) and (18) we obtain for Θ the formula (12).

Smith[5] replaced the sphere problem by the Bethe-lattice problem, a procedure that is generally speaking not a neutral operation. This enabled him to obtain $\Theta(T)$ in the form of a curve with a kink at $T=\Theta$. In particular, at $T>\Theta$ it was found that

$$\chi = \frac{n_m \mu^2 S^2}{3T} . \tag{19}$$

It can be stated that this formula is valid also for the true sphere problem close enough to the transition point.

The heat capacity at Θ has in practice no singularities. This follows from the fact that contributions to the heat capacity is made only by bonds of the type $T(\Theta)$, and these are due mainly to peripheral spins that join the clusters. The singular part of the heat capacity is due to bonds that must become closed in order to form an infinite cluster. The number of such bonds is inversely proportional to the cluster dimension, which is of the order of $|\beta - \beta_c|\gamma$, where $\phi = 1.69 \pm 0.3$.[7] This yields for the heat capacity a singular part of the order of

$$C_{\text{sing}} \sim \left(\frac{T-\Theta}{\Theta} \right)^{\gamma} (n_m^{1/3} l)^{\gamma} n_m \tag{20}$$

This part vanishes together with its derivative at $T=\Theta$, and at $T-\Theta>\Theta$ it is small in comparison with the nonsingular part obtained above.

For the same reasons, $\rho(T)$ has likewise no singularities.

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