Phase diagram of quasi-zero-dimensional electron-hole system

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We consider phase transitions with formation of an exciton condensate in two-dimensional electron-hole systems in strong transverse magnetic fields. The phase diagram of the system is constructed for densities at which only the lower Landau level is filled, when the particle motion is "zero-dimensional."

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1. We predict in this paper phase transitions with formation of an exciton condensate in quasi-two-dimensional electron-hole (*e–h*) systems (quantized films, layered semiconductors, etc.) in strong transverse magnetic fields H. The properties of the obtained excitonic phase differ qualitatively from the properties of those investigated in three-dimensional *e–h* systems in strong H.[1-3] The phase diagram of the system is calculated at $r_H=(c/eH)^{1/2}\ll a_0(a_0=\min(a_e,a_h)$, $a_{e,h}$ are the effective Bohr radii of e and h, and $\hbar=1$) and at a density $n\lesssim 1/2r_H^2$–when all the particles are at the lower Landau level and their motion is effectively zero-dimensional. The phase diagram consists of regions I and III–tenuous and dense *e–h* plasma–and an exciton phase in region II (Fig. 1). The transitons I–II and I–III are of second order. In the zeroth order in r_H/a_0, the thermodynamic characteristics of the exciton phase coincide with the characteristics of a state made up of an *e–h* liquid drop and *e–h* gas,[4] which are calculated with Maxwell's rule taken into account. Therefore the transition of a tenuous *e–h* gas (I) into a continuous *e–h* liquid (III) can be treated as a first-order transition or as two successive second-order transitions. The situation is analogous to

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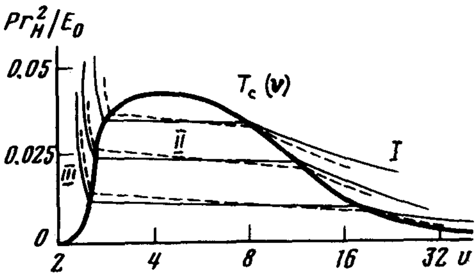

FIG. 1.

ideal Bose-gas condensation, which can be regarded either as a second-order phase transition[5] or as first-order transition.[6] The physical meaning of this analogy lies in the fact that the multipole moments of the "zero-dimensional" excitons are equal to zero (see below), and the interaction between them appears only when corrections $\sim r_H/a_0$ are taken into account. Nonetheless, these phases are microscopically different, and allowance for the corrections makes the thermodynamic properties also different.

2. In the problem of one two-dimensional exciton in a magnetic field at $r_H \ll a_0$, the Coulomb interaction can be regarded as a perturbation (it is important that, in contrast to the three-dimensional case, the spectrum of the unperturbed particles is exclusively discrete here). The Coulomb interaction (in the c.m.s.) is diagonal in the angular quantum number m, i.e., it does not lift the degeneracy in m. Therefore, the binding energy of the exciton is determined in first order in the interaction and is equal to $E_0=(\pi/2)^{1/2}e^2/\epsilon r_H$ (in the three-dimensional anisotropic case the exciton binding energy is also $\sim\sqrt{H}$[7]) and coincides with the binding energy of the quasi-zero-dimensional e–h liquid.[4] At the lower Landau level there are no excited exciton states, which can be due only to transitions to the next levels. The wave functions of the ground states of the particles do not depend on their masses in the zeroth order in r_H/a_0. Therefore the multipole moments of the exciton are equal to zero, and the interaction between the excitons appears only in the next orders in r_H/a_0. We note that at $T\neq 0$, in the low-density limit, the incoherent excitons disintegrate (the analog of the Saha law).

3. The transition to the exciton phase is described by an analog of the Gor'kov equations, since the non-ladder diagrams are small of the order of $(r_H/a_0)^n$. From these equations we obtain in the usual manner an equation for the order parameter Δ_0 connected with pairing at the lower level:

$$\Delta_0 = \frac{E_0}{2}\sum_{n=0}^{\infty} I_n^2 \frac{\Delta_0}{(\xi_n^2 + I_n^2\Delta_0^2)^{1/2}} \operatorname{th}\frac{(\xi_n^2 + I_n^2\Delta_0^2)^{1/2}}{2T}. \tag{1}$$

Here $\xi_n = n\omega_H + \epsilon_n - \mu$, ω_H is the cyclotron frequency, $\epsilon_n = 2E_0 I_n v^{-1}$ are the obtained renormalizations of the Landau levels, $v=I/\pi n\, r_H^2$ is the specific two-dimensional volume in r_H^2 units, μ is the chemical potential, $I_0=1$, and $I_n=(2n-1)!!/(2n)!!$.

The terms with $n \geqslant 1$ in (1) make a contribution that is small in r_H/a_0. From (1) we obtain the dependence of the transition temperature T_c on v:[1)]

$$T_c = E_0(1 - 4/v)/2\ln(v/2 - 1)\,, \tag{2}$$

which has a maximum $T_{max} = E_0/4$ at $v = 4$. At a given temperature $T < E_0/4$ two first-order transitions take place in the system: at $v = v_1(T)$ from the tenuous e–h plasma (region I in Fig. 1) into the exciton state (region II), and at $v = v_2(T)$ from the exciton phase into the condensed e–h plasma (region III) ($v_{1,2}(T)$ are the critical volumes determined from (2), and $1/v_1 + 1/v_2 = \frac{1}{2}$). At $T = 0$, in the zero-density level, the state of the system (just as in the three-dimensional case[8]) is that of an exciton condensate, since $v_1(0) = \infty$. The free energy of the system at $T < T_c$ (i.e., in the region II), is

$$F(v, T) = (v/v_1(T))F_{eh}(v_1, T) - E_0(1 - v/v_1(T)), \tag{3}$$

where F_{eh} is the free energy of the e–h plasma (see[4]). In this region we have $(\partial P/\partial v)_T = 0$ (P is the pressure). The isotherms $P_T(v)$ of the exciton phase, shown by the thin lines in Fig. 1, coincide exactly with the isotherms of the e–h plasma, linearized in accordance with Maxwell's rule in the liquid–gas region (which coincides with the region II). Region II can therefore be regarded either as a region where some of the e and h dropped out into the exciton condensate between the second-order phase-transition points, or as a region of liquid–gas coexistence on the line of the $v_1(T)$–$v_2(T)$ first-order transition. The isotherms coincide because there is no interaction (accurate to r_H/a_0) between the excitons, so that the e–h liquid can be regarded as a result of "sticking together" of the excitons.

4. Despite the equality of the thermodynamic characteristics, the single-phase exciton state and the two-phase drop state are, obviously, microscopically different, a fact reflected, for example, in the spectra of the elementary excitations. Allowance for corrections in r_H/a_0 eliminates also the thermodynamic equivalence of these states. Thus, when the corrections are taken into account we have in the exciton state $(\partial P/\partial v)_T < 0$ (the isotherms of the exciton state with allowance for the corrections are shown dashed in the figure). At low density ($v \sim v_1(T)$) the exciton phase is energywise favored, whereas at $v \sim v_2(T)$ the two-phase drop state is favored. The energy difference in this case is merely $\lesssim 0.1(r_H/a_0)E_0$, so that one cannot exclude the possibility that effects not accounted-for here (impurities, surface, etc.) can, without hardly changing each of the states, move them more substantially apart.

We note in conclusion that the exciton state can be distinguished from the drop state by means of the cyclotron-resonance spectra. In the drop state two lines appear (at $m_e = m_h$), corresponding to the liquid and the gas, shifted by an amount $E_0 v_{1,2}^{-1}$ from ω_H, while in the homogeneous excitonic state we have one line shifted by an amount $\sim E_0$.

[1)]Analogous results for the critical temperature of the exciton pairing in a system with spatially separated e and h were obtained independently in[9]. We thank the authors for their preprint.

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