Heating of parametrically turbulent plasma

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(Submitted 26 March 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **27**, No. 9, 504–507 (5 May 1978)

An expression for the rate of heating of a turbulent plasma is obtained under conditions of strong parametric coupling between the excited waves.

PACS numbers: 52.50.Gj, 52.50.Jm, 52.35.Py, 52.35.Ra

Parametric instability of a plasma in a strong electromagnetic field is extensively used for anomalous heating of a plasma. A characteristic of the rate of absorption of radiation energy by the plasma is the effective collision frequency $\nu_{\rm eff}$.[1] The product of $\nu_{\rm eff}$ by the pump-field energy density $E_0^2/4\pi$ yields the power $Q=\nu_{\rm eff}E_0^2/4\pi$ absorbed by the plasma in a unit volume. In the weak-field limit (or, equivalently, in the limit of large detuning from resonance), $\nu_{\rm eff}$ was determined in weak-turbulence theory.[2] In the limit of very strong fields (or very small detunings from resonance), $\nu_{\rm eff}$ was obtained in a strong-turbulence theory.[3] Neither of these limiting cases, however, is suitable for the description of the phenomena occurring in the broad intermediate region, which is of practical interest and in which, in particular, laser plasma is presently being investigated.

To determine $\nu_{\rm eff}$ (or Q) we consider the parametric excitation of electron Langmuir oscillations with wave number k_d and growth rate $\gamma\equiv\gamma(E_0,k_d)$. We assume that k_d is large enough, so that the dispersion correction to the Langmuir-wave frequency $\omega_{Le}(k_d r_D)^2$ exceeds the ion-sound frequency $\omega_{Li}k_d r_D$ i.e., $k_d r_D>\omega_{Li}/\omega_{Le}$. This inequality holds as a rule for the parametric instabilities of practical interest.

Under the indicated conditions, the principal nonlinear process that limits the exponential growth of the amplitude of parametrically excited Langmuir oscillations is their secondary parametric instability to excitation of other Langmuir and low-frequency waves. A quasistationary state is produced in the buildup region if the growth rates γ and γ_k of the primary and secondary parametric instabilities are equal, $\gamma_{k=k_d}=\gamma$. This equation alone is sufficient to determine the plasma-wave field intensity $E_{l\gamma}$ in the buildup region, and by the same token the power density $Q=\gamma E_{l\gamma}^2/4\pi$ pumped into the plasma.

In a nonisothermal plasma, under conditions of weak coupling of the parametrically excited waves

$$\gamma < \omega_{Li}k_d r_D \tag{1}$$

the growth rate of the secondary parametric instability $\gamma_k\simeq(1/2\sqrt{2})\times(\omega_{Le}\omega_{Li}k_d r_D)^{1/2}E_{l\gamma}/\sqrt{4\pi n_e T_e}$ increases linearly with the plasma-wave amplitude in the buildup region. In this case Q is proportional to the cube of the growth rate of the primary instability[2]

$$Q \approx 8n_eT_e\gamma^3/(\omega_{Le}\omega_{Li}\ k_dr_D).\tag{2}$$

In an isothermal plasma, when the damping decrement γ_s of the ion-sound is of the order of the sound frequency, $\omega_{Li}k_dr_D \sim \gamma_s$, we have under the weak-coupling conditions (1) $\gamma_k \approx (\omega_{Le}/8)(E_{ly}^2/4\pi n_eT_e)$. The power input then increases in proportion to the square of the growth rate[4]

$$Q \approx 8n_eT_e\gamma^2/\omega_{Le}.$$

These known expressions for Q change qualitatively under conditions of strong parametric coupling of the waves, when the inequality (1) is violated. The spectrum of the low-frequency oscillations is then strongly distorted[1]

$$\omega_k \sim \gamma_k \sim \omega_{Le}^{1/3}\omega_{Li}^{2/3}(k_dr_D)^{2/3}(E_{l\gamma}^2/4\pi n_eT_e)^{1/3} \propto E_{l\gamma}^{2/3}.$$

The power pumped into the plasma is then proportional to the fourth power of the growth rate

$$Q \approx n_eT_e\gamma^4/(\omega_{Le}\ \omega_{Li}^2 k_d^2r_D^2).\tag{3}$$

The direction of energy transfer under the strong-coupling conditions remains the same as in case (1) (from the higher to the lower frequencies), provided that γ is small compared with the dispersion correction to the Langmuir-wave frequency

$$\omega_{Li}k_dr_D < \gamma < \omega_{Le}(k_dr_D)^2.\tag{4}$$

In this case there is no aperiodic instability in the wave-buildup zone and the restructuring of the spectra retains its decay character [although the frequency ω_k does not coincide with the ion-sound frequency under the conditions (4)]. Aperiodic instability can arise in the case (4) at a certain $k_a < k_d$ for which $\gamma_{k_a} \approx \omega_{Le}(k_ar_D)^2$.

At still higher values of the growth rate γ, when the inequality (4) is violated, the aperiodic instability appears directly in the buildup region. In this case $\gamma_k \approx \omega_{Li}E_{ly}/\sqrt{4\pi n_eT_e}$ and Q is again proportional to the cube of the growth rate[3]

$$Q \approx n_eT_e\gamma^3/\omega_{Li}^2.\tag{5}$$

Thus, formula (3) derived above provides a direct transition between the cases of weak fields (large detunings) (2) and strong fields (small detunings) (5).

The presented expressions for Q determine the rate of pump-energy transfer to the plasma at arbitrary growth rates γ of the detuning k_d in the quasistationary state, when all the power Q pumped into the plasma oscillations is subsequently absorbed by the plasma particles. This absorption is due either to electron-ion collisions or to transfer of the Langmuir-oscillation energy from the long to the short waves (as a result of aperiodic parametric instability) with subsequent dissipation in the Cerenkov-damping region. From the point of view of plasma heating, the first regime, of collision absorption, is preferable. Let us discuss it in greater detail.

Under weak-coupling conditions (1), according to[2], the collision regime of parametric turbulence is realized at $\nu_{ei} > \gamma\omega_{Li}(\omega_{Le} k_d r_D)^{-1}$. For a dense plasma with $T_e \sim 1$ keV, heated by laser radiation of wavelength of the order of one micron, this inequality is violated later than (1). The collision regime is therefore possible under the strong-coupling conditons (4), provided that there is no secondary aperiodic instability, i.e., that there is no noise in the long-wave region $k \lesssim k_a$. This means that the total change $N\gamma$ of the Langmuir-wave frequency (N is the number of secondary decays and γ is the change of the frequency of the Langmuir wave as a result of one decay under the conditions (4)) should be smaller than the dispersion increment to the frequency of the excited wave $\omega_{Le}(k_d r_D)^2$. Since each decay act causes the growth rate of the Langmuir wave to decrease by an amount $\sim\nu_{ei}$, it is clear that $N \sim \gamma/\nu_{ei}$. Therefore the collisional absorption regime is realized at

$$\nu_{ei} > \gamma^2 / \omega_{Le} (k_d r_D)^2. \tag{6}$$

This inequality, while more stringent than (4), still covers the entire region of heating-radiation flux densities of practical interest under conditions of a laser plasma. For parametric instability in the critical-density region it follows from (6) that

$$q < 10^{14} A\sqrt{z}\, \lambda^{-7/2} T_e^{-5/4} \text{ W/cm}^2,$$

where T_e is in keV, λ is the wavelength of the heating radiation in microns, and A and z are the atomic weight and the charge of the target material.

If this inequality is violated, formula (3) for Q remains in force, but the power input to the plasma waves goes no longer into heating but into acceleration of the electrons.

We note in conclusion that the formulas presented for the power absorbed as a result of the instability are valid not only for parametric instabilities but also for others, such as stream instabilities, provided only that $k_d r_D > \omega_{Li}/\omega_{Le}$. For parametric instabilities in a laser plasma, this inequality is practically always satisfied by virtue of the large characteristic wave number $k_d \approx r_D^{-1} \times \ln^{-1/2}(\omega_{Le}/\nu_{ei}) \approx 0.3 r_D^{-1}$ of the excited oscillations.

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