# Stabilization of tearing instability in stationary flow of a plasma

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It is shown that the spreading of a plasma over a current sheath (CS) may turn out to be a strong stabilizing factor if the growth rate of the tearing instability (TI), calculated for the static case, is small in comparison with the relative rate of expansion of the CS.

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The idea of reconnection of magnetic force lines and the concept of the CS play an important role in the physics of cosmic and laboratory plasma.[1-5] Investigation of the TI,[6,7] which changes the topology of the magnetic field, is therefore of exceptional interest.

Although the linear theory predicts the existence of TI in a wide range, both in collisionless plasma[6] and in the MHD approximation,[7] laboratory experiments[8,9] show that it has a threshold. A broad CS exists for an appreciable time in comparison with the TI growth rate, and only then does the formation of the tear take place. The use of various types of mechanisms of stabilizing the TI is essential also for application to the CS in the earth's magnetosphere and in models of solar flares, where the free energy of the magnetic field increases with the width of the CS.

Stabilization of the TI during the linear stage by plasma outside the sheath was considered in[10,11]. The nonlinear distortion of the particle trajectories was investigated in[12]. In the linear approximation, the TI is modified when account is taken of the magnetic field normal to the CS[13] or of the plasma flow into the layer.[14] The study of these models constitutes an attempt to investigate real plasma streams characterized by generally complicated nonstationary velocities and magnetic fields.

We estimate below the spreading of the plasma over a CS. We assume that the CS lies in the $(x, z)$ plane; the current is directed along the $z$ axis, the magnetic field is given by $\mathbf{H}=H_0\,\mathrm{sign}(y)\mathbf{e}_x$. Let a plasma flow into the CS along the $y$ axis and spread along the $x$ axis. It appears that this picture corresponds to stationary convection in the tail of the magnetosphere,[4] as well as to the state of the CS prior to the flare in the solar-flare model.[2,3,10] We assume the velocity $v_x$ to be of the form[1)]

$$v_x=xh, \tag{1}$$

where $h$ is the relative rate of spreading of the layer and depends generally speaking on the time ($h>0$).

For simplicity we calculate the TI for an expanding CS with a thickness $L$ that is small in comparison with the Larmor radii $r_{H\alpha}$ ($\alpha=e,i$) of the particles. If $kL \ll m_e/m_i$; $r_{He}/L \gg (m_i/m_e)^{1/2}$ then the hydrodynamic description is valid. Here $k$ is the wave number of the perturbations.[2)] Linearizing the equations of two-fluid hydrodynamics about the initial state $v=xh+v_{1x}$, $n=n_0+n_1$, $A_z=A_0+A_1$, we obtain:

$$\dot{v}_{1x} + hxv'_{1x} + hv_{1x} = -\frac{e}{m_i c} u_0 \dot{A}_1 , \tag{2}$$

$$\dot{n}_1 + hxn'_1 + n_0 v'_{1x} + hn_1 = 0. \tag{3}$$

The prime denotes differentiation with respect to the coordinates and a dot will denote differentiation with respect to time; $n$ is the density, $u_0$ is the drift velocity, and $A_1$ is the vector-potential perturbation and satisfies the equation

$$\frac{\partial^2 A_1}{\partial x^2} + \frac{\partial^2 A_1}{\partial y^2} = \frac{4\pi e L}{c} \delta(y)\left(\frac{en_0}{m_e c} A_1 + n_1 u_0\right). \tag{4}$$

Following the theory of gravitational instability in an expanding universe,[16] we represent the spatial dependence of the perturbed quantities in the form

$$f_1(x, t) = \bar{f}(t)\exp(iqx/a(t))f_1 = (v_{1x}, n_1, A_1).$$

$q$ is the dimensionless wave number; $a(t)$ is a certain function of the time with the dimension of length. Solving Eq. (4) relative to $A_1$ we obtain

$$\dot{\bar{v}} + h\bar{v} + i\frac{qx}{a}\left(h - \frac{\dot{a}}{a}\right)\bar{v} = iq\frac{m_e}{m_i}\frac{u_0^2}{q\lambda_e + a}\bar{n}, \tag{5}$$

$$\dot{\bar{n}} + h\bar{n} + i\frac{qx}{a}\left(h - \frac{\dot{a}}{a}\right)\bar{n} = -i\frac{q}{a}\bar{v}. \tag{6}$$

$\lambda_e = m_e c^2/2\pi n_0 e^2 L$. We choose $a(t) = b\exp[\int h(t)\,dt]$. We obtain the solution of (5) and (6) by assuming $q\lambda_e < a(t=0) = b$. Namely

$$\begin{pmatrix} v_{1x} \\ n_1 \end{pmatrix} = C_1\begin{pmatrix} 1/a(t) \\ iq/\gamma_0 a(t) \end{pmatrix}\exp\left(\gamma_0 b\int\frac{dt}{a(t)} + i\frac{qx}{a(t)}\right)$$
$$+ C_2\begin{pmatrix} 1/a(t) \\ -iq/\gamma_0 a(t) \end{pmatrix}\exp\left(-\gamma_0 b\int\frac{dt}{a(t)} + i\frac{qx}{a(t)}\right). \tag{7}$$

$\gamma_0 = qu_0(m_e/m_i)^{1/2}/b$ is the growth rate of the TI in the static case. It is seen that as $t\to\infty$ the solution is asymptotically stable. If $\gamma_0 < h$, then the perturbations decrease monotonically with time. At $\gamma_0 > h$ the amplitude of the perturbations first increases and decreases only afterwards. The maximum increase is of the order of the ratio $(\gamma_0/h)^{\gamma_0/h}$, i.e., the stabilization is ineffective at $\gamma_0 \gg h$. The condition for the suppression of the TI

$$\gamma_0 < h \tag{8}$$

can be obtained in the general case on the basis of simple estimates by comparing the kinetic energy of the expansion of the CS and the energy released in the course of development of the TI, which tends to gather the plasma into individual clusters.

According to [17,18], the rate of outflow of the plasma from the layer is of the order of the Alfven velocity $v_A = H_0/(4\pi n_0 m_i)^{1/2}$, which is of the order of $v_{Ti}$. From this we can obtain the value of the "Hubble constant" $h \approx v_A/b$. It follows from (8) that the TI can be suppressed if the growth rate $\gamma_0$ is relatively small, as is the case in the MHD approximation when the plasma has high conductivity and the magnetic Reynolds number is $R_m = 4\pi\sigma L v_A/c^2 \gg 1$. In this case $\gamma_0 \approx (v_A/L) R_m^{-3/5} (kL)^{-2/5}$.[6] The stabilization condition is

$$kL > (b/L)^{5/2} R_m^{-3/2}. \tag{9}$$

Since $kL < 1$ (see[6]), it follows that the CS is stable if $b < LR_m^{3/7}$. For a diffuse CS of collisionless plasma ($L \gg r_{H\alpha}$) the growth rate is $\gamma_0 \sim kv_{Ti}(m_e/m_i)^{1/2}(r_{He}/L)^{3/2}$. The condition (8) is equivalent to

$$kL < \frac{L}{b}\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{L}{r_{He}}\right)^{3/2}. \tag{10}$$

If $b < L(m_e/m_i)^{1/2}(L/r_{He})^{3/2}$, then there is no TI in the sheath. The instability can develop when the width of the CS exceeds a certain critical value determined by (9) and (10).

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[1)]Results of a numerical simulation of the formation of CS are given in [15] and show that the law governing the spreading of the plasma over the layer is close to (1).
[2)]In the present article we do not consider the connection between the TI and the stream instabilities.


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