Possible mechanism of self-focusing in a plasma

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The propagation of a light beam in a plasma is accompanied by the flow of current due to the action of the light pressure. The resultant magnetic field influences the refractive index of the plasma and can lead to self-focusing. We derive expressions for the threshold self-focusing power and for the focal length.

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The propagation of a light beam in a plasma can be accompanied by the flow of current due to the action of the light pressure.[1,2] The magnetic field resulting from the flow of the current will influence the refractive index of the plasma, and this can lead to self-focusing. We obtain here expressions for the threshold self-focusing power and for the focal length.

The current density j_s due to the action of the light pressure is determined from the equations of motion of the electrons and ions and from the momentum conservation law written down under the assumption that the ponderomotive forces due to the gradients of the light-beam intensity are small:

$$mN\frac{d\mathbf{v}_e}{dt} = -NeE_s - \frac{Ne}{c}[\mathbf{v}_e\times\mathbf{B}_s] - \nabla p_e - (\mathbf{v}_e - \mathbf{v}_i)\nu mN;\quad \frac{1}{c}[\mathbf{j}\times\mathbf{B}_s] = \frac{\mu\mathbf{s}}{c}$$

$$MNz^{-1}\frac{d\mathbf{v}_i}{dt} = NeE_s + \frac{Ne}{c}[\mathbf{v}_i\times\mathbf{B}_s] - \nabla p_i + (\mathbf{v}_e - \mathbf{v}_i)\nu mN;\quad \mathbf{j} = Ne(\mathbf{v}_i - \mathbf{v}_e), \tag{1}$$

where $\mathbf{E}_s$ and $\mathbf{B}_s$ are the electric and magnetic fields of the light wave, ν is the frequency of the electron-ion collisions, $\mathbf{s}$ is the Poynting vector, μ is the absorption coefficient, N is the electron density, $\mathbf{v}$ is the velocity, and p is the pressure. The subscripts e and i refer to the electrons and ions. We introduce a cylindrical coordinate system (r,ϕ,z) whose z axis coincides with the optical axis of the beam. Solving the system (1) and averaging over a time segment much longer than the period of the light wave we can write for the projections $j_z\equiv\langle j_z\rangle_t$, $u_{ez}\equiv\langle v_{ez}\rangle_t$, and $u_{iz}\equiv\langle v_{iz}\rangle_t$

$$MNz^{-1}u_{iz} = \frac{\mu M}{cM^*}\int_0^t [W(t') - W(t-t')e^{-\nu t'}]dt' - \frac{M}{M^*}\int_0^t \frac{\partial(p_e+p_i)}{\partial z}dt + \frac{M}{\nu M^*}\frac{\partial p_e}{\partial z}, \tag{2}$$

$$mNu_{ez} = \frac{\mu zm}{cM^*} \int_0^t [W(t') + \frac{M}{zm} W(t-t') e^{-\nu t'}] dt' - \frac{zm}{M^*} \int_0^t \frac{\partial (p_e + p_i)}{\partial z} dt$$

$$- \frac{M}{\nu M^*} \frac{\partial p_e}{\partial z}, \tag{3}$$

$$j_z = Ne(u_{iz} - u_{ez}) = -\frac{\mu e}{mc} \int_0^t W(t-t') e^{-\nu t'} dt' + \frac{e}{m\nu} \frac{\partial p_e}{\partial z}, \tag{4}$$

where $M^* = M + zm$ and $W \equiv \langle s \rangle_t$ is the intensity of the laser radiation. From (2) and (3) we see that the momentum of the electromagnetic radiation ($\mu W/c$) is transferred almost entirely (accurate to terms zm/M) to the ions in the presence of electron-ion collisions. Equation (4) shows that a current due to the light pressure can flow in the plasma. The first term in the right-hand side of (4) is j_s, while the second term describes the diffusion contribution to j_z. It follows from (4) that $j_s \to 0$ as $\mu \to 0$. The relaxation time of j_s is equal to ν^{-1}. In the case when W varies slowly enough with time ($W^{-1}\dot{W} \ll \nu$), the expression for j_s takes the form $j_s = -\mu e W/mc\nu$. Using this expression for an axially symmetrical beam with radius ρ and assuming that W does not depend on r (at $r \leqslant \rho$) we obtain for the magnetic field B the relation $B = 2\pi e \mu W r / mc^2 \nu$, which is valid at $r \leqslant \rho$. The direction of the magnetic field B coincides with the direction of the magnetic field of the current flowing along the z axis counter to the laser beam, and is thus always perpendicular to the wave normal. In this case birefringence is produced in the beam and we can write for the refractive indices n and the absorption coefficients μ in the case $\mathbf{E}_s \perp \mathbf{B}$ the relations[3]

$$n^2 = 1 - \frac{\alpha(1-\alpha)}{1-\alpha-\beta}; \qquad \mu = \frac{\nu\alpha[(1-\alpha)^2 + \beta]}{nc[1-\alpha-\beta]^2}, \tag{5}$$

where $\alpha = \omega_0^2/\omega^2 \approx N/N_{cr}$; $\beta = \omega_B^2/\omega^2$; ω_0 is the electron plasma frequency, ω_B is the Larmor frequency, and N_{cr} is the critical density. The expressions presented for n and μ are valid if the inequality $1-\alpha-\beta \gg \nu/\omega$ is satisfied. Assuming in addition $\beta \ll (1-\alpha)^2$, we simplify the expressions for n and μ:

$$\mu = \frac{\nu\alpha}{c\sqrt{1-\alpha}}; \qquad n = \sqrt{1-\alpha} - \frac{\alpha\beta}{2(1-\alpha)^{3/2}}. \tag{6}$$

Since B reaches a maximum near the beam boundary and $B=0$ on the beam axis, n increases in the case $\mathbf{E}_s \perp \mathbf{B}$ towards the beam center, and conditions favoring self-focusing are produced. We note that in this case the connection between the change Δn of the refractive index and the power density W has a nonlocal character, in contrast to the well investigated cubic media with nonlinearity of the Kerr type. The problem will hereafter be solved for laser radiation with circular polarization. In this case the vector $\mathbf{E}_s$ at any point over the beam cross section can be resolved into two components of equal magnitude, one parallel and the other perpendicular to the magnetic field. The birefringence of the light should lead to formation of two coaxial light beams of equal intensity, in one of which (S_1) the vector $\mathbf{E}_s$ is everywhere parallel to the magnetic field, and in the other (S_2) it is perpendicular to the field. The beam S_2 is capable of self-focusing, whereas the beam S_1 diverges as the result of diffraction. To estimate the self-focusing threshold power P_{cr} we neglect the contribution of the light pressure of the beam S_1, for the diameters of the beams S_1 and S_2 can differ greatly in the course of the propagation, both as a result of the self-focusing of the beam S_2 and as a result of the diffraction divergence of S_1. Following[4], we estimate the threshold power by equating the half-value angle of the diffraction divergence of the beam S_2 $\Delta\phi_{\text{diffr}} = 1.22\lambda_0/4\rho n$ to the critical glancing angle in total internal reflection $\Delta\phi_{\text{gl}} = \sqrt{\alpha\beta}/(1-\alpha)$. Expressing β in terms of W, we determine P_{cr} from the relation

$$P_{cr} \approx \pi\rho^2 W = 0.96\,\frac{\nu m^2 c^4}{e^2\mu}\sqrt{\frac{1-\alpha}{\alpha}}\,. \tag{7}$$

We write down the equation of the trajectory of the light rays in the form $Rr''_{z^2} = -(1+r'^2_z)^{3/2}$, where $R = n(\partial n/\partial r)^{-1}$ is the radius of curvature of the light rays. Using Eq. (6), we obtain $R = \gamma^2/r$, where $\gamma = m^2c^3\nu\omega(1-\alpha)/2\pi e^2\mu W\sqrt{\alpha}$. Assuming $r'^2_z \ll 1(\rho_0^2 \ll f^2)$, we can write $r''_{z^2} + \gamma^{-2}r = 0$, from which it follows that f can be estimated from the relation $f = (\pi/2)\gamma(0)$. Recognizing that $f \to \infty$ as $P \to P_{cr}$ and substituting the values of the constants, we write

$$f = 0.82\,\frac{\omega}{c}\,\frac{\rho_0^2\sqrt{1-\alpha}}{P_0/P_{cr}-1}\,. \tag{8}$$

With increasing z, the power $P = P_0 e^{-\mu z}$ decreases because of the inverse-bremsstrahlung absorption of the light. If the self-focusing is not to be stopped by the light absorption, it is necessary to satisfy the inequality $f \leqslant \mu^{-1}\ln P_0/P_{cr}$, which, given α, P_0, and ν, imposes the following limitation on the initial beam radius:

$$\rho_0^2 \leqslant \frac{1.22c^2}{\alpha\omega\nu}(P_0/P_{cr}-1)\ln(P_0/P_{cr}). \tag{9}$$

We note that at the beam boundary the magnetic field increases with decreasing z in accordance with the expression

$$B(\rho) = \frac{2\pi e\mu W}{\nu m c^2}\rho \sim \frac{\exp(-\mu z)}{\rho(z)}$$

as $\rho \to 0$ ($z \to f$). However, with increasing B the condition $\beta \ll (1-\alpha)^2$ is violated and the absorption coefficient μ begins to increase, thus limiting the subsequent increase of the magnetic field. The maximum value of B can be estimated from the relation $\beta \approx (1-\alpha)^2$.

We present a numerical example: for $P_0 = 10^{11}$ W; $\nu = 10^{12}$ sec; $\omega = 1.8\times10^{15}$ sec^{-1}; $\alpha = N/N_{cr} = 0.9$, calculation by formulas (7)–(9) yields

$$P_{cr} \approx 10^9\ \text{W};\quad \rho_0 \lesssim 2\cdot10^{-2}\ \text{cm};\quad f \approx 4\cdot10^{-2}\ \text{cm}.$$

Under these conditions, the magnetic field reaches a value 10 MG. The expressions for f and P_{cr} were obtained under the condition that W does not vary over the beam cross section. Similar calculations for a beam with a Gaussian profile of the power density $W = W_0 e^{-r^2/\rho^2}$ lead to the following expressions for the threshold power P'_{cr} and for the value of the focal length f'.

$$P'_{cr} \approx 1.6\, P_{cr}\,; \quad f' \approx f\,(1 + r^2/\rho^2)\,.$$

A more correct calculation of the self-focusing process (in particular, the structure of the focal region) and of the maximum values of the magnetic field for the proposed mechanism calls for the use of a computer. Since the considered effect depends strongly on the proximity of N to N_{cr}, the possibility of observing it in a laser plasma is determined to a considerable degree by the gradients of the electron density near N_{cr}. We note at the same time that the foregoing analysis is valid when the wavelength of the laser radiation in the plasma is less than the diameter of the light channel. When N_{cr} is approached, this condition is violated, a fact that must be taken into consideration.

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[1]G.A. Askar'yan, M.S. Rabinovich, A.D. Smirnova, and V.B. Studenov, Pis'ma Zh. Eksp. Teor. Fiz. **5**, 116 (1967) [JETP Lett. **5**, 93 (1967)].

[2]P.P. Pashikin and A.N. Prokhorov, Pis'ma Zh. Eksp. Teor. Fiz. **26**, 687 (1977) [JETP Lett. **26**, 526 (1977)].

[3]V.L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in Plasma), Fizmatgiz, 1960, p. 141.

[4]R.Y. Chiao, E. Garmire, and C.H. Townes, Phys. Rev. Lett. **13**, 479 (1964).