# Superconductivity produced in multivalley semimetals by Coulomb interaction between the electrons

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The feasibility and conditions of superconductivity in multivalley semimetals are considered with account taken of only the Coulomb interaction between the electrons. It is shown that, depending on the density, Cooper pairs with different orbital angular momenta can be produced.

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We consider a model of a multivalley semimetal with large number of electron ($\nu_e$) and hole ($\nu_h$) valleys. We assumed that the valleys are spherical in shape, $\nu_e=\nu_h=\nu \gg 1$, and that the electron and whole masses are equal ($m_e=m_h=m$), the centers of the different valleys are located at one point, and there are no transitions between different valleys. The spectra of the electronic transitions of the electron and hole types then are respectively of the form:

$$\epsilon_e(\mathbf{P}) = \frac{\mathbf{p}^2 - p_F^2}{2m}\,;\quad \epsilon_h(\mathbf{p}) = \frac{p_F^2 - \mathbf{p}^2}{2m}$$

($p_F$ is the Fermi momentum). We shall henceforth take into account only the Coulomb interaction between the electrons. We chose this system of units $e^2/\epsilon=\hbar=m=1$ ($\epsilon$ is the static permittivity of the medium). The energy of the ground state of such a semimetal was calculated in[1], where it was shown that it has a minimum at

$$n = n_0 = \frac{2^{24/5}}{3^{8/5}\pi^{7/5}}\,\frac{\nu^{8/5}}{[\Gamma(\frac{1}{4})]^{24/5}},$$

with $n^{1/4} \gg P_F$ at densities $n\sim n_0$. Let us analyze the diagrams for the total vertex $\Gamma_{\alpha\alpha}(p_1p_3P)=\Gamma(p_1p_3P)$, where $\alpha=1$ and 2 are the indices of the electron and hole valleys, respectively, and $P=(\Omega,\mathbf{P}=0)$ is the total momentum. The screened Coulomb interaction

$$U(k) = \frac{V(\mathbf{k})}{1 - 2\nu\Pi_0(k)V(\mathbf{k})}\quad \left( V(\mathbf{k}) = \frac{4\pi}{|\mathbf{k}|^2}\,;\ \Pi_0 = \Pi_0^{(e)} = \Pi_0^{(h)}\right.$$

(is the bare polarization operator) reaches a maximum at $|\mathbf{k}|\sim n^{1/4} \gg p_F, \omega\sim n^{1/2} \gg \epsilon_F$. The renormalized interaction, which acts as the bare interaction in the Cooper diagram, is therefore substantially different from $U(k)$. In the diagrams for the total vertex, which can be cut by two parallel lines with identical directions into two unconnected parts (we call such diagrams weakly-connected), it is convenient to subdivide

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the region of integration with respect to the momentum corresponding to these parallel lines into three subregions: $|\epsilon(\mathbf{q})|<\xi, \xi<|\epsilon(\mathbf{q})|<\epsilon_1, |\epsilon(\mathbf{q})|>\epsilon_1$, where $\Omega \ll \xi \ll \epsilon_1 \sim \epsilon_F$. We shall regard as irreducible with respect to the Cooper channel only those weakly connected diagrams in which integration with respect to the momentum of at least one pair of parallel lines joining two unconnected parts of the diagram is carried out in the first subregion: $|\epsilon(\mathbf{q})|<\xi$. The remaining diagrams would be regarded as irreducible and designated $\gamma^{\xi}_{\alpha\alpha}(p_1p_3)=\gamma^{\xi}(p_1p_3)$. Diagrams reducible with respect to the Cooper channel would be proportional to $\ln(\xi/\Omega)$. The irreducible diagrams will consist of some that are proportional to powers of $\ln(\epsilon_1/\xi)$, and some that contain no logarithmic singularities at all. The sum of these diagrams will be designated $\lambda^{\xi}_{\alpha\alpha}(p_1p_3)=\lambda^{\xi}(p_1p_3)$ and $\gamma_{\alpha\beta}(p_1p_3)$, respectively. We note that logarithmic singularities in the dielectric electron-hole channel are disregarded for reasons that would be stated below. The main sequence of the diagrams for $\gamma_{\alpha\beta}(p_1p_3)$ is shown in Fig. 1. The integration in Fig. 1(b) is carried in the subregion $|\epsilon(\mathbf{q})|>\epsilon_1$, to avoid a logarithmic singularity. The main subsequence of the diagrams for $\lambda^{\xi}(p_1p_3)$ constitutes Cooper ladders in which the bare interaction is represented by $\gamma(p_1p_3)$ and the integration with respect to the momentum of parallel lines of the ladder is carried out in the subregion $\xi<|\epsilon(q)|<\epsilon_1$. In this case

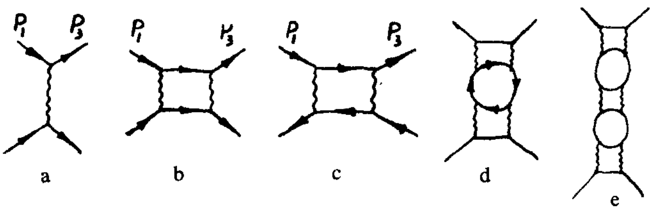

FIG. 1. Main sequence of diagrams for $\gamma_{\alpha\beta}(p_1p_3)$.

$\gamma^{\xi}=\gamma+\lambda^{\xi}$. The sum of the two diagrams 1(b) and 1(c) will be designated $\lambda_{\alpha\beta}$. Just as in[2], the main contribution to the integral for $\lambda_{\alpha\beta}$ is made by the momentum and frequency $|\mathbf{q}|\sim n^{1/4}\gg p_F$, $\omega\sim n^{1/2}\gg\epsilon_F$, so that $\lambda_{\alpha\beta}$ can be regarded as independent of the external momenta if the latter are in the vicinity of the Fermi surface. The expression for $\lambda_{\alpha\beta}$ takes the form

$$\lambda_{\alpha\beta}=\frac{(-1)^{\alpha+\beta+1}}{2n^2}\int\frac{d\omega\, d^3\mathbf{q}}{(2\pi)^4}\left[\frac{V(\mathbf{q})\,\Pi_0(\omega q)}{1-2\nu V(\mathbf{q})\,\Pi_0(\omega q)}\right]^2=\frac{(-1)^{\alpha+\beta+1}\pi^{3/4}}{n^{3/4}[\Gamma(\frac{1}{4})]^2}.$$

We see therefore that $\lambda_{11}=\lambda_{22}=-\lambda_{12}=\lambda<0, |\lambda|\ll 1$. The summation of the diagram for $\gamma_{\alpha\beta}(p_1p_3)$ is carried out just as in[2]

$$\gamma_{\alpha\beta}=\frac{V(\mathbf{p}_1-\mathbf{p}_3)}{1-2\nu\Pi_0(p_1-p_3)\,V(\mathbf{p}_1-\mathbf{p}_3)}+\frac{\lambda_{\alpha\beta}}{1-2\nu\lambda\Pi_0(p_1p_3)},$$

but in contrast to[2], $\gamma_{\alpha\beta}$ has no pole at $n\geqslant n_0$. The diagrams for $\Gamma(p_1p_3P)$ have the structure of Cooper ladders, in which the bare interaction is represented by $\gamma^{\xi}(p_1p_3)$, and the integration over the momenta of parallel lines is carried out in the subregion

$|\epsilon(q)| < \xi$. It is therefore clear that in $\gamma^{\xi}(p_1 p_3)$ we can put $|\mathbf{p}_1| = |\mathbf{p}_3| = p_F$ and $\omega_1 = \omega_3 = 0$ (accurate to $\xi/\epsilon_F \ll 1$). For this reason $\gamma^{\xi}(p_1 p_3)$ will depend only on the angle $\theta$ between $\mathbf{p}_1$ and $\mathbf{p}_3$. Recognizing that $\nu \Pi_0(t) \gg p_F^2$, we obtain

$$\gamma_{11} = \gamma_{22} = \gamma(t) = \frac{1}{-2\nu\Pi_0(t)} + \frac{\lambda}{1 - 2\nu\lambda\Pi_0(t)}$$

where $t = \sin(\theta/2)$. Using the expression for $\lambda$ and the expression for $\Pi_0(t)$,[3] which is valid at $\omega \ll k V_F = 2p_F^2 t$, i.e., at $t \gg \omega/\epsilon_F \sim \xi/\epsilon_F$ we obtain

$$2\nu\Pi_0(t) = -\left(\frac{n_0}{n}\right)^{5/12} \frac{3}{8}\left[1 + \frac{1 - t^2}{2t}\ln\frac{1 + t}{1 - t}\right].$$

Since we integrate over the angles in the diagrams for the total vertex, and since the region of small angles $\theta \ll 1$ makes a small contribution to the integral, the restriction on the angle is inessential. Plots of $\gamma(t)$ at two values of the density, $n = n_0$ and $n = 2.2n_0$, are shown in Fig. 2. Since $\gamma(t)$, meaning also, $\gamma^{\xi}(t)$, depends substantially on the angle $\theta$ between $\mathbf{p}_1$ and $\mathbf{p}_3$, it is convenient to expand, in the expression for the total vertex, in Legendre polynominals $P_l(\cos\theta)$. If $\ln(\epsilon_F/\xi) \ll \ln(\epsilon_F/\Omega)$ and $|\gamma_l| \ll 1$, we can easily determine[4] $\gamma_l^{\xi}$, which is found to be $\lambda_l^{\xi} \ll \gamma_l$, meaning that $\gamma_l^{\xi} \approx \gamma_l$. The ex-

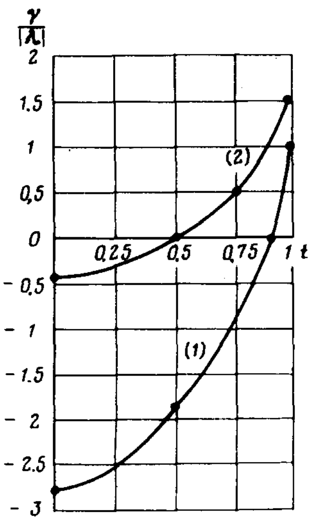


FIG. 2. Plots of $\gamma(t)$: 1—at $n = n_0$, 2—at $n = 2.2n_0$.

pression for the $l$-th harmonic of the total vertex $\Gamma_l$ then becomes

$$\Gamma_l(\Omega) = \gamma_l^{\xi}\left[1 + \frac{p_F}{2\pi^2(2l+1)}\gamma_l^{\xi}\ln\frac{2\xi}{\Omega}\right]^{-1} \approx \gamma_l\left[1 + \frac{p_F}{2\pi^2(2l+1)}\gamma_l\ln\frac{2\epsilon_F}{\Omega}\right]^{-1}.$$

It follows that $\Gamma_l(\Omega)$ has a pole if $\gamma_l < 0$. Furthermore, singlet pairing occurs if $\gamma_l < 0$ starting with even $l$, and triplet pairing starting with odd $l$.[3] Numerical calculations show that at $n=n_0$ we have $\gamma_0<0$ and $|\gamma_0|>|\gamma_1|$ and singlet pairing is realized at $l=0$. At $n=2.2n_0$ we have $\gamma_0>0$ and $\gamma_1<0$, meaning that triplet pairing with $l=1$ is realized.

We disregarded here in the dielectric electron-hole channel the logarithmic singularities that are typical of an excitonic dielectric.[5] We assume that the hole Fermi surface deviates little from a sphere. The degree $\delta\epsilon$ of non-coincidence of the electron and hole Fermi surfaces is assumed to satisfy the condition $T_c^* \ll \delta\epsilon \ll \epsilon_F$, where $T_c^*$ is the temperature of the transition to the state of the excitonic dielectric in the model with exactly spherical bands. Then the logarithmic singularities vanish in the electron-hole channel and remain in the Cooper channel. Moreover, since $\delta\epsilon \ll \epsilon_F$, this influences little the expression for the effective interaction $\gamma(t)$, and the results of the paper remain the same as before.

We note also that if the centers of the valleys are at different points in the Brillouin cell, this does not influence the effective interaction and the pairing of the electrons from one valley.

The transition to the superconducting state on account of the Coulomb interaction is posssible also in layered semimetal with one electron band and one hole band. This, however, is the subject of a seperate study.

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