Shubnikov–de Haas oscillations in the cleavage plane of a germanium bicrystal

B. M. Vul and E. I. Zavaritskaya

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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Shubnikov–de Haas oscillations were observed at helium temperature and in constant magnetic fields ~ 100 kOe in the cleavage plane of germanium crystals with inclination angle $\theta \approx 25°$. As a result of these investigations we determined the concentration and the relaxation times of the light and heavy holes in a two-dimensional conducting layer, information of importance for the determination of the cause of the high electric conductivity of such a layer.

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It was established in[1] that the p-type conductivity layers produced on the growth boundary of germanium bicrystals with inclination angles $30° \geqslant \theta \geqslant 10°$, can be regarded, from the point of view of the electric conductivity, as a two-dimensional medium. In a transverse magnetic field, the electron motion in such a medium is fully quantized, and it was of interest to investigate the Shubnikov–de Haas oscillations under these conditions.

We measured the resistivity $\rho(H)$ and the Hall coefficient $R(H)$ of germanium bicrystals with angle $\theta \approx 25°$ in constant magnetic fields up to 150 kOe applied perpendicular to the cleavage plane.

The results of the measurements for one such sample are shown in Figs. 1 and 2. It is seen from the presented data that at $H > 50$ kOe oscillations are observed in the $\rho(H)$ and $R(H)$ plots; the amplitude of the oscillations increases with the magnetic field intensity and reaches approximately 2% at $H = 100$ kOe, whereas the total change of $\Delta\rho/\rho(0)$ and $\Delta R/R(0)$ in the field range $0 \lesssim H \lesssim 150$ kOe is approximately 10%.

It is known that the period of the Shubnikov–de Haas oscillations is

$$\Delta\left(\frac{1}{H}\right) = \frac{e\hbar}{mcE_F}, \tag{1}$$

where E_F is the Fermi energy at $H = 0$ and m is the carrier effective mass.

In the two-dimensional case and for a quadratic dispersion law, the Fermi energy is $E_F = (\pi\hbar^2/m)n$ and

$$\Delta\left(\frac{1}{H}\right) = \frac{e}{\pi c\hbar}\frac{1}{n} = \frac{4.85 \cdot 10^6}{n}, \tag{2}$$

where n is the carrier density.

For our sample, as seen from the measurement results, the oscillation is $\Delta(1/H)=6.7\times10^{-6}$ Oe^{-1}, and accordingly the hole density is $n\approx7\times10^{11}$ cm^{-2}, whereas it follows from Hall measurements that in each of the two hole layers adjacent to the cleavage plane the hole density is $n_H\approx7\times10^{12}$ cm^{-2}, i.e., ten times larger.

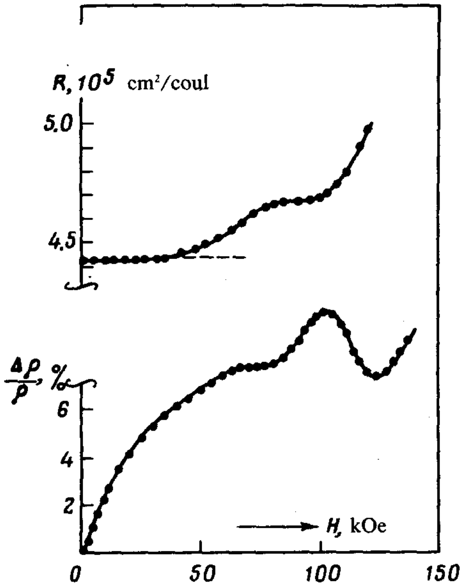

FIG. 1. Resistivity $\rho(H)$ and Hall coefficient $R(H)$ vs magnetic-field intensity H at $T=4.2$ K.

Obviously, the difference between the quantities n and n_H is due to singularities of the energy structure of the valence band of germanium, which consists, if no account is taken of the spin-orbit splitting, of bands of light and heavy holes whose effective masses $m_l=0.042m_0$ and $m_h=0.36m_0$[2] differ by an approximate factor of 10. Inasmuch as in the two-dimensional model in the case of degeneracy the hole density is $n=(E_F/\pi\hbar^2)m$, and E_F is the same for both types of hole, the ratio of the densities n_h and n_l of the heavy and light holes is $n_h/n_l=m_h/m_l\approx10$.

It is easy to show that in our case the Hall effect is determined predominantly by the heavy holes. Actually, in the presence of two types of carrier, the Hall coefficient is

$$R_{H\to0}=\frac{1}{e}\,\frac{n_l\mu_l^2+n_h\mu_h^2}{(n_l\mu_l+n_h\mu_h)^2}\,, \tag{3}$$

where n_l and n_h are the densities of the two types of carrier, μ_l and μ_h are their mobilities at $H=0$, and τ_l and τ_h are the relaxation time. In bicrystals at low temperatures, the predominant mechanism is scattering by charge centers whose number in our case reaches $N_i=1.4\times10^{13}$ cm^{-2}. In this case[3]

$$\frac{1}{\tau}=N_i\,v\,2\pi\left\{\frac{e^2}{\kappa m v^2}\right\}f(x), \tag{4}$$

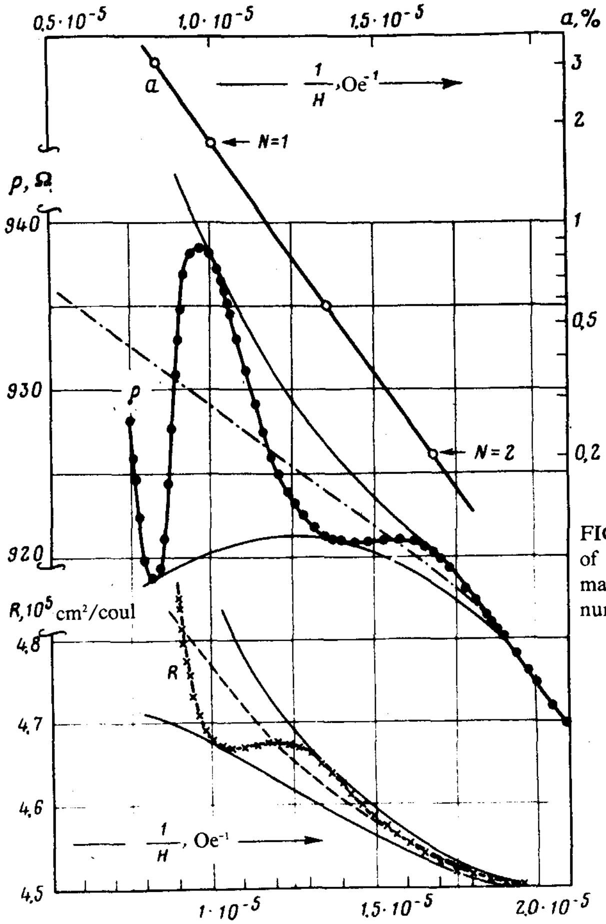

FIG. 2. Plots of ρ, R, and the amplitude a of their oscillations against the reciprocal magnetic field intensity (N is the quantum number).

where κ is the dielectric constant, v is the carrier velocity, and $f(x)\approx 1$ and can be neglected in our case.

Since the heavy and light holes have the same energy and are scattered by the same centers, it follows from (4) that

$$\frac{\tau_l}{\tau_h} \cong \frac{v_h}{v_l} \quad \text{and} \quad \frac{\mu_l}{\mu_h} \cong \left(\frac{m_h}{m_l}\right)^{1/2}. \tag{5}$$

Using the fact that $n_h/n_l = m_h/m_l$, we obtain from (3) that $R_{H\to 0}\approx 1.1/en_h$, i.e., R is determined mainly by the value of n_h. In this approximation we can also estimate the mobility $\mu_h \cong R/\rho$ of the heavy holes and their relaxation time $\tau_h = m_h\mu_h/e$. For our samples, $\mu_h = 580$ cm/V-sec and $\tau_h \approx 10^{-13}$ sec.

In contrast to the Hall measurements, in Shubnikov–de Haas oscillations the light holes play the major role. Actually, the distance $\hbar\omega_c = (e\hbar H/c)(1/m)$ between the Landau levels is inversely proportional to the carrier mass. At $H = 100$ kOe we have for the light holes $(\hbar\omega_c)_l = 28$ meV and for the heavy holes $(h\omega_c)_h$ is about 3 meV. The order of magnitude of the smearing ΔE of the levels can be estimated from the uncertainty relation $\Delta E \sim \hbar/\tau$. The relaxation time of the light holes, calculated according to[4] from the damping of the oscillation amplitude, is $\tau_l \approx 3\times 10^{-14}$ sec, and their mobility $u_l \approx 1400$ cm^2/V-sec is approximately three times the mobility of the heavy holes, in good agreement with the relation (5) used by us above. The smearing of the Landau levels for the light holes is

$$(\Delta E)_l \sim \frac{\hbar}{\tau_l} \approx 19 \text{ meV} < (\hbar\omega_c)_l \quad \text{at} \quad H = 100 \text{ kOe}.$$

The relaxation time of the heavy holes, as already mentioned, is $\tau_h \approx 10^{-13}$ sec and, accordingly, the smearing of their levels is

$$(\Delta E)_h \sim \frac{\hbar}{\tau_h} \approx 6 \text{ meV} > (\hbar\omega_c)_h \quad \text{at} \quad H = 100 \text{ kOe}.$$

If we introduce the Dingle temperature,[5] then the level smearing will be somewhat less, but enough to prevent observation of oscillations on heavy holes. This follows also from the relation[6]

$$\left(N + \frac{1}{2}\right) = \frac{\dfrac{1}{H^N_{\max}}}{\Delta\left(\dfrac{1}{H}\right)}, \tag{6}$$

where N is the quantum number and $H^N_{\max}$ is the field intensity corresponding to the maximum of the oscillation.

From the data shown in Fig. 2 it follows that $N=1$ and $H_1 = 10^5$ Oe and $N=2$ at $H_2 \approx 6\times 10^4$ Oe, whereas for heavy holes these levels can be observed only in fields on the order of a million oersteds.

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[1] B.M. Vul, E.I. Zavaritskaya, Yu.A. Bashkirov, and V.M. Vinogradov, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 204 (1977) [JETP Lett. **25**, 187 (1977)].
[2] B. Lax, H.J. Zeiger, and R.N. Dexter, Physica (Utrecht) **20**, 818 (1954).
[3] R. Mansfield, Proc. Phys. Soc. London S. B **69**, part 1, 76 (1956).
[4] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **36**, 959 (1974); **37**, 1233 (1974).
[5] A.B. Dingle, Proc. Roy Soc. London Ser. A **211**, 517 (1952).
[6] I.M. Tsidil'kovskiĭ, Élektrony i dyrki v poluprovodnikakh (Electrons and Holes in Semiconductors), Nauka, 1972.